

## **LESSON 15**

*Format*

- [MATHEMATICAL EXPRESSIONS REQUIRING RUNOVERS](#)
  - [Mathematical Units](#)
  - [Calculus](#)

[Answers to Practice Material](#)

### **LESSON PREVIEW**

In this lesson we discuss what to do when a mathematical expression will not fit on one braille line. Rule 26.2 of the Nemeth code gives basic instructions for selecting runover sites, but application of the rule can be daunting given the wide variety of layouts encountered in a transcription. Lesson 15 explores many typical scenarios. The examples serve as a good review of other aspects of the Nemeth code.

**MATHEMATICAL EXPRESSIONS REQUIRING RUNOVERS**

[NC 26.2]

**15.1 Review**

Above all else, a mathematical expression must not be divided between lines if it fits one braille line within the current margins. Before considering division options, check whether the math will fit on one line ...

... if you place related code switch indicators on a different line, will the math fit on one line?

... if you start the math on a new line following its related identifier, will it fit on one line?

... if you move the math to a line that does not contain a page number, will it fit on one line?

Here is a review of items that must not be divided and a summary of runover rules already covered in the lessons.

**15.1.1 Symbols to Keep Together.** The components of the following symbols must not be divided between braille lines.

- A symbol of operation using plus and minus (Lesson 5)
- A symbol of comparison compounded vertically or horizontally (Lesson 5)
- A shape symbol with structural or interior modification (Lesson 11)
- A keystroke construction (Lesson 11)
- Superposed symbols (Lesson 13)
- Tally marks belonging to the same group (Lesson 13)

**15.1.2 Expressions to Keep Together.** The following expressions must not be divided between braille lines, even if divided in print.

- A hyphenated expression of which one component requires Nemeth. (Lesson 2)
- An abbreviation and its related numeral or letter. (Lessons 3 and 4)
- A shape symbol and its name (numeral, letter, or sequence of letters). (Lesson 11)
- The components of an expression modified according to the five-step rule. (Lesson 12)
- A function name (or its abbreviated form) and its argument. (Lesson 14)
- A two-part function name. (Lesson 14)

**15.1.3** When a long expression does not fit on the braille line, use as much of the first line as possible before dividing at an appropriate location. A hyphen is not inserted, with one exception (a long numeral). The new line begins in the runover cell of the current format. Here is a list of runover rules already studied:

- **Long Numeral.** If a long numeral will not fit on one line, divide after a comma if a comma is present. A hyphen is inserted at the point of division. If the numeral does not contain a comma, the hyphen may be inserted after any digit. The numeric indicator is restated before the first digit of the continuation of the numeral on the next line. (Lesson 1)
- **Enclosed List.** If an enclosed list will not fit on one line, divide after a comma used to separate the items. (Lesson 4)
- **Linked Expression.** If a linked expression will not fit on one line, divide before a link. It is not necessary to divide at every link unless the layout follows that of a nested linked expression. (Lesson 8)
- **A Series of Keystrokes.** If a series of keystrokes will not fit on one line, division is made after any item in the keystroke string but not within a keystroke. (Lesson 11)
- **Individual Mathematical Expressions.** When two or more math expressions occur between the same code switch indicators, the line may wrap at the space between the expressions, even if the entire Nemeth portion could fit on one line. (Lesson 1)

## A SYSTEMATIC APPROACH

### 15.2 Mathematical Units

The braille transcriber has only 40 cells available on a line at most – as few as 30 cells in some layouts. We often encounter a mathematical expression that will not fit on the current line. The margins in place at the time should not be changed in order to accommodate a long expression. Instead, the math will need to be divided.

A long or complicated mathematical expression can be organized into a series of mathematical units by following the procedures presented in this lesson. When the transcriber applies these principles, the reader is able to mentally reassemble the expression. On the other hand, a poorly divided expression will hinder the reader's understanding of the mathematics. Shrewd application of these guidelines can be properly rendered even if the transcriber is unfamiliar with the particular mathematics.

We will now examine how to choose runover sites in order to keep the following mathematical units intact:

- the anchor in a linked expression
- a link in a linked expression
- a fraction or, if unavoidable, its numerator or its denominator
- a grouped expression
- a base and its exponent; a subscript and its related item
- a radical expression
- a modified expression or, if unavoidable, a modifier within the modified expression

When choosing a division site, start with step a.

**Step a** Divide before a comparison sign on the baseline. See [Section 15.3](#).  
(The comparison sign cannot be within a mathematical unit.)

If there is no such comparison sign or if there is still not room, apply step b.

**Step b** Divide before an operation sign on the baseline. See [Section 15.4](#).  
(The operation sign cannot be within a mathematical unit.)

If there is no such operation sign or if there is still not room, choose from step c or step d.

**Step c** Divide before a mathematical unit. See [Section 15.5](#).

**Step d** Divide after a termination indicator. See [Section 15.6](#).

*Each step in the systematic approach will now be examined. Take your time analyzing each example. Isolated examples without narrative do not include code switch indicators in order to focus on layout.*

### 15.3 Step a: Divide Before a Comparison Sign on the Baseline

As you learned in Lesson 8, a linked expression is comprised of an anchor followed by one or more links and division is made at a link (before the comparison symbol) when the entire expression will not fit on one braille line.

Keep these points in mind as you study the examples in this section.

- Determine whether the expression can fit on one line using tactics given in [Section 15.1](#).
- If the math will not fit, use as much of the line as possible before dividing.
- The comparison sign at which the division is made must be on the baseline of writing.
- The link begins in the appropriate cell according to the current format. ([Example 15-1](#))  
*Note:* The print copy may divide after a comparison sign, but the braille transcription divides *before* the comparison symbol.
- Even if the anchor consists of only one letter or number, if the link will not fit on the line with the anchor, the line is divided after the anchor. ([Example 15-2](#))
- If the expression contains more than one link, it is not necessary to divide at every link. ([Example 15-3](#))

*Exceptions:* A nested linked expression is divided at every link (see Example 8-37 in Lesson 8); if the anchor or any link is divided, divide at every link (to be covered in [Section 15.4.2](#)).

- When a line begins with a comparison symbol, the transition to a new braille line terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines. ([Example 15-4-b](#))
- In itemized formats, if an anchor will not fit on the line with its identifier but it fits on the next line starting in the runover cell, put it there in order to keep the anchor intact. The identifier will then be the only item on the first line. (Examples [15-4-a](#) and [15-5](#))







15.3.1 A Sign of Comparison Within a Mathematical Unit

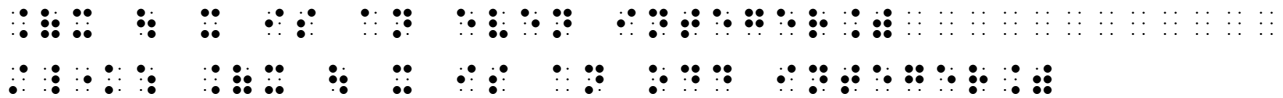
Unless unavoidable, a sign of comparison within a mathematical unit is not a suitable division site. (See the list of mathematical units in [Section 15.2](#).)

Example 15-6

THE LINKED EXPRESSION REQUIRES 62 CELLS

(embedded)

$$\{x \mid x \text{ is an even integer}\} \not\subseteq \{x \mid x \text{ is an odd integer}\}$$



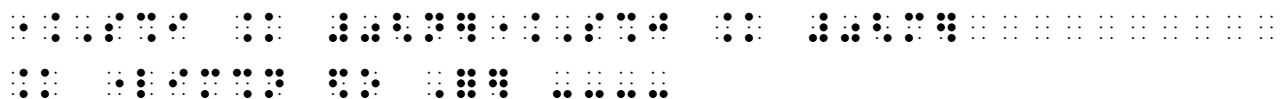
*A grouped expression is a mathematical unit. Within each grouping the vertical bar meaning "such that" is a comparison sign but, since the grouping will fit on one line, it is not divided. The preferred division site in this linked expression is the comparison sign between the groupings (the negated bar under inclusion meaning "is not a subset of").*

Example 15-7

THE LINKED EXPRESSION REQUIRES 52 CELLS

(embedded)

$$\sum_{i=0}^n \sum_{j=0}^m = \lim_{n \rightarrow \infty} \text{---}$$



*A modifier is a mathematical unit. The comparison sign within each of these modifiers is not a suitable division site (equals sign, arrow). The preferred division site in this linked expression is the comparison sign printed on the baseline of writing (the equals sign before the function name).*

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**PRACTICE 15A**  
**LINKED EXPRESSIONS**

*Instructions:* Keep the following points in mind as you select runover sites. (1) Before dividing an expression, try placing the switch indicators on a different line. (2) Before dividing an expression, try placing the identifier on a different line. (3) If the entire expression will not fit on the line, divide before a comparison sign on the baseline.

A. Solve the linear inequalities.

i.  $(x + \frac{10}{3})(x + \frac{19}{3}) > (3x + \frac{46}{3})(\frac{x}{3} + 1)$

ii.  $\frac{2x}{3} - 3 > \frac{16x}{21} - \frac{13}{3} - \frac{2x}{15}$

iii.  $(a - 1)^2 - (a - 7)(a - 3) < 2a + 0.8$

B.  $2 \times 423 = (2 \times 400) + (2 \times 20) + (2 \times 3) = 800 + 40 + 6 = 846$

**Binomial coefficients** get their name because they are the *coefficients* in the expansion of a *binomial*:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}.$$

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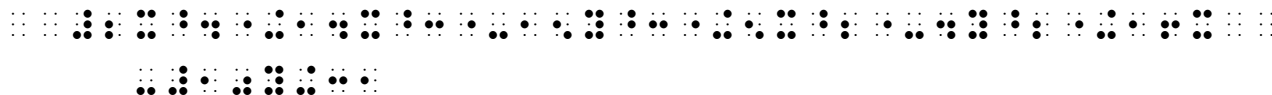


Example 15-9

THE EXPRESSION REQUIRES 43 CELLS

(displayed)

$$2x^4 + 14x^3 - 15y^3 + 5x^2 - 4y^2 + 16x - 10y + 31$$



Line 1: The displayed expression begins in cell 3. All but the last two terms fit on this line.

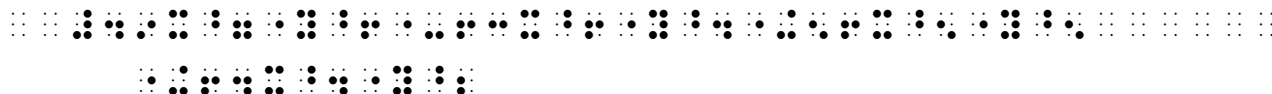
Line 2: The runover line begins in cell 5 with an operation symbol. A numeral preceded by a minus symbol requires a numeric indicator when the minus symbol begins a braille line.

Example 15-10

THE EXPRESSION REQUIRES 43 CELLS

(displayed)

$$49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2$$



Line 1: The displayed expression begins in cell 3. All but the last term fits on this line.

Line 2: The runover line begins with the baseline indicator which precedes the operation symbol (the plus symbol).

## 15.4.1 A Sign of Operation within a Mathematical Unit

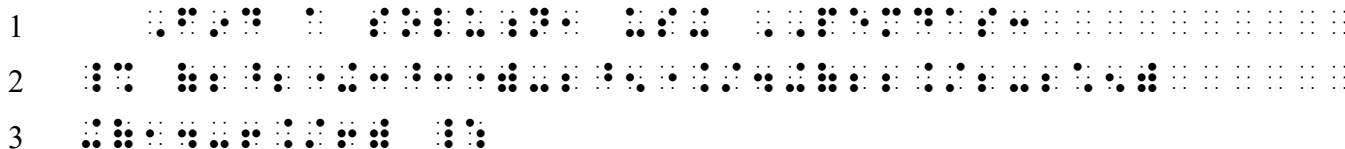
Unless unavoidable, an operation sign within a mathematical unit is not a suitable division site. (See the list of mathematical units in [Section 15.2](#).)

Example 15-11

THE EXPRESSION REQUIRES 42 CELLS

(embedded)

Find a solution, using PEMDAS:  $(2^2 + 3^3) - 2^5 \div 4 + (22 \div 2 - 2 \cdot 5) + (14 - 6 \div 6)$



Line 1: A grouped expression is a mathematical unit. Although the first addend will fit on line 1, that plus sign is within a grouped expression and so is not a suitable division site.

Line 2: The opening switch is placed on the same line as the beginning of the embedded mathematical expression.

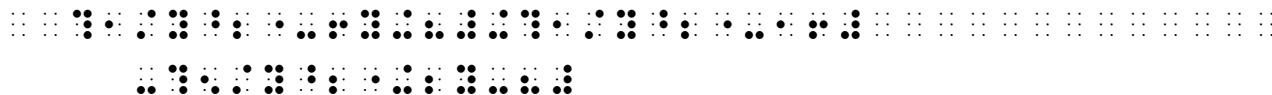
Line 3: The operation sign outside of the grouped expression is the preferred division site. Line 3 begins with a plus symbol that is situated between two groupings.

Example 15-12

THE EXPRESSION REQUIRES 39 CELLS

(displayed)

$$\frac{1}{y^2 - 6y + 8} + \frac{1}{y^2 - 16} - \frac{5}{y^2 + 2y - 8}$$



Line 1: Starting in cell 3, the entire math expression will not fit on one line.

Lines 1-2: A fraction is a mathematical unit. Each fraction is kept intact on one line. The operation signs within the denominators are not suitable division sites.

Line 2: The operation sign on the baseline of writing is the preferred division site. Line 2 begins with the minus symbol before the last fraction.

15.4.2 **Further Considerations Within a Linked Expression.** Review the bullet points in [Section 15.3](#) (step a). If an anchor or a link will not fit on the line and must be divided, further rules apply.

i. **A linked expression with only one link.** If a division must be made *within* the anchor or its link, a division must still be made before the link. In other words, step a is applied first, then step b. (Examples [15-13](#) and [15-14](#).)

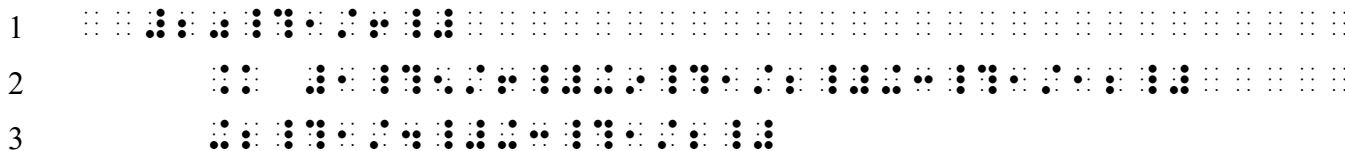
ii. **A linked expression with two or more links.** In a linked expression with more than one link, if a division must be made *within* the anchor or any link, a division must still be made before each link. The resulting transcription maintains an orderly representation of mathematical units. ([Example 15-16](#))

Example 15-13

ANCHOR: 10 CELLS; LINK: 49 CELLS

(displayed)

$$20\frac{1}{6} = 1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2}$$



Line 1: The anchor is on line 1.


Line 2: The runover line begins with the comparison symbol—the equals sign (step a). The link does not fit entirely on line 2.


Line 3: The second runover line begins with an operation symbol—a plus sign (step b).


Example 15-14ANCHOR: 48 CELLS; LINK: 5 CELLS

(itemized)

5.  $144 \text{ ft}^2 + 17 \text{ ft}^2 + 112 \text{ ft}^2 + 15 \text{ ft}^2 - 131 \text{ ft}^2 = N$

1 

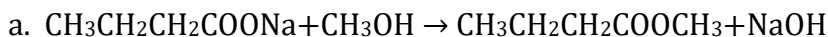
2 

3 


*Line 1: After checking that the anchor will not fit entirely on line 2, it begins here.**Line 2: The anchor is divided before the plus sign of the fourth term (step b).**Line 3: The linked expression is divided before the equals sign (step a). The link must not be placed on line 2 even though it will fit because step b has been applied—that is, because the anchor has been divided.*Example 15-15ANCHOR: 34 CELLS; LINK: 37 CELLS

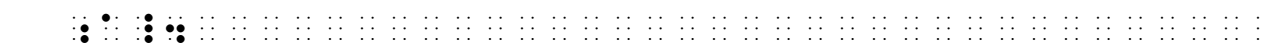
(itemized)


1. Consider the following chemical equations.

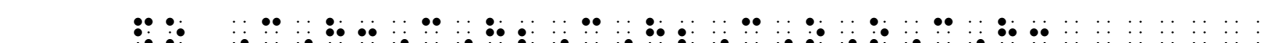



b. ...


1 

2 

3 

4 

5 

6 

*Lines 2-3: The anchor does not fit on the line with the identifier, but it will fit entirely on the next line. It begins in the runover position for subdivisions (cell 5).**Line 4: Line 4 begins with the comparison symbol (right-pointing arrow, step a). The link will not fit entirely on this line. It is divided before the operation sign (plus sign, step b).**Line 5: Because the subscript (3) at the end of line 4 does not require a subscript indicator, there is no baseline indicator needed before the plus symbol that begins on line 5.*







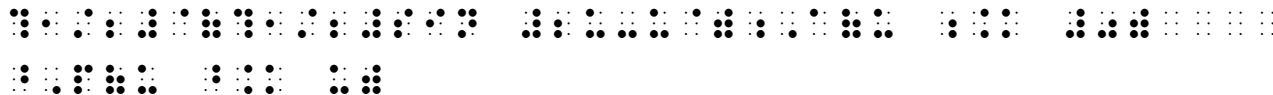


Example 15-22

THE EXPRESSION REQUIRES 48 CELLS

(embedded)

$$\frac{1}{2} \left[ \frac{1}{2} \sin 2u - u \right]_{A(u=0)}^{P(u=u)}$$



The best option is to keep grouped expression between brackets undivided (18 cells). The subscript is transcribed first, according to Nemeth rules for simultaneous superscripts and subscripts (Lesson 6). The subscript (13 cells) fits on the first line, but the superscript (12 cells) does not. The superscript begins the runover line, starting with the superscript indicator.

15.5.4 **Further Considerations Within a Linked Expression.** Review the bullet points in Sections [15.3](#) (step a) and [15.4](#) (step b). If a step c division must be made within an anchor or within a link, work your way through these steps.

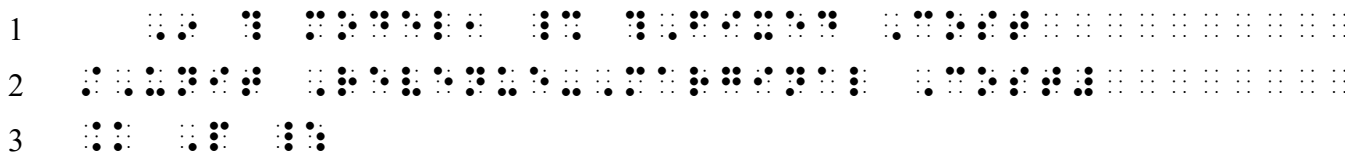
- First, divide before the link, starting the runover line with the comparison symbol (step a). If there is more than one link, start a new line with each link.
- Within the anchor or link that will not fit on the line, divide before an operation sign on the baseline, if present (step b).
- Finally, divide before a mathematical unit (step c).

Example 15-23

ANCHOR: 44 CELLS; LINK: 5 CELLS

(embedded linked expression)

In this model,  $\frac{\text{Fixed Cost}}{\text{Unit Revenue} - \text{Marginal Cost}} = P$



The anchor (a fraction) needs to be divided. Working through the steps,  
 Step a (comparison sign on the baseline): The first division is made before the link. This will become line 3 which begins with the equals symbol.  
 Step b does not apply because the operation sign (minus sign) is not on the baseline.  
 Step c (mathematical unit): Since the fraction will not fit on one line, division is made before the fraction line. The numerator fits on line 1. The denominator fits on line 2, beginning with the fraction line and ending with the closing fraction indicator.  
 Line 3: Even though the link will fit on line 2, division before the comparison sign is required (step a).





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**PRACTICE 15C**

*Instructions:* Note that the ellipsis in item c. is on the baseline of writing. Review Section 8.22.4 in Lesson 8 regarding the layout of problem 2.

1. Compute and/or simplify.

a. 
$$\frac{\left(\frac{3}{2}\right) \times \left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{1 \times 2 \times 3}$$

b. 
$$\frac{\frac{dx}{dt} \frac{d^2y}{dt^2} - \frac{d^2x}{dt^2} \frac{dy}{dt}}{(dx/dt)^3}$$

c.  $(\pm)a_{1i_1} a_{2i_2} a_{3i_3} a_{4i_4} \dots a_{ni_n}$

2. 
$$\frac{x^2+2}{x(2x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{2x^2+1} + \frac{Dx+E}{(2x^2+1)^2}$$

$$= \frac{A(4x^4+4x^2+1)+B(2x^4+x^2)+C(2x^3+x)+Dx^2+Ex}{x(2x^2+1)^2}$$


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**PRACTICE 15D**

1. Multiply these four polynomials:  $(4x^9y)(7x^8y^2)(5x^7y^3)(6x^6y^4)$ .

2. 
$$\pm \left( \frac{a^{\frac{2}{3}} + a^{\frac{1}{3}} b^{\frac{1}{3}} + b^{\frac{2}{3}}}{3} \right)^{\frac{3}{2}}$$

3.  $(16x^4 + 8x^3y + 4x^2y^2 + 2xy^3 + y^4)(2x - y) = 32x^5 - y^5$

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**15.6 Step d: Divide After a Termination Indicator**

If a suitable division site cannot be found and if a termination indicator is present, begin a new line after the termination indicator.

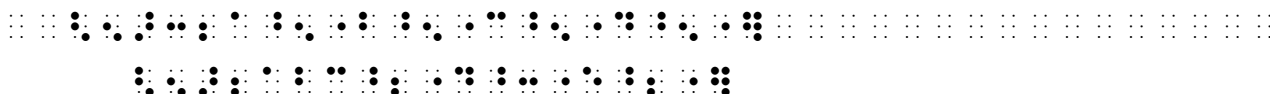
15.6.1 **A Radical Expression is a Unit.** Division may be made after the termination indicator that ends a radical expression. If the entire radical expression will not fit on the line, apply division strategies to the radicand.

Example 15-28

THE EXPRESSION REQUIRES 41 CELLS

(displayed)

$$\sqrt[5]{32a^5b^5c^5d^5} \sqrt[5]{2abc^2d^3e^2}$$



Division is made after the termination indicator of the first radical expression.

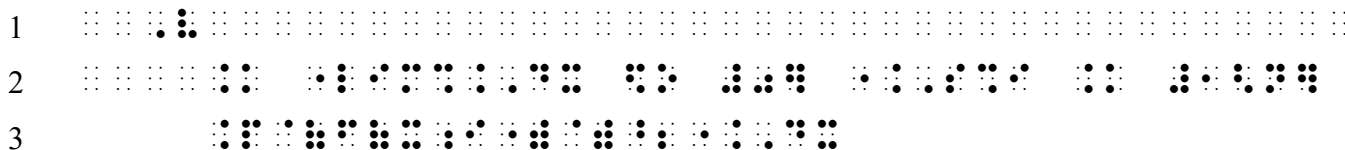
15.6.2 **A Modified Expression is a Unit.** Division may be made after the termination indicator that ends an expression which is modified according to the five-step rule of modification. (Lesson 12) If the entire modified expression will not fit on the line it is acceptable to divide before the directly over or the directly under indicator. That strategy will be illustrated in the next section with [Example 15-34](#).

Example 15-29

ANCHOR: 2 CELLS; LINK: 55 CELLS

(displayed)

$$V = \lim_{\Delta x \rightarrow 0} \sum_{i=1}^n \pi [f(x_i)]^2 \Delta x$$



Line 2: The first division is made before the link (step a). The link does not fit on one line. Division is made after the termination indicator which ends the second modified expression.

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**PRACTICE 15E**

*Note: In item 2, the expression is displayed.*

1.  $\sqrt[3]{x^1} \sqrt[6]{x^2} \sqrt[12]{x^3} \sqrt[24]{x^4} \sqrt[48]{x^5} \dots$

2. The general solution for one root of the cubic equation is

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}.$$

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**PRACTICE 15H**

*Instructions:* Transcribe this expression as if it were embedded within narrative, beginning with an opening Nemeth Code indicator in cell 1. Show two different ways to divide this expression.

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}$$

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## SUMMARY

Keep these basic points in mind as you consider the layout of a long mathematical expression. First, determine whether the expression can fit on one line using the tactics given in [Section 15.1](#). If the math must be divided, use as much of the line as possible before dividing. The following division strategies were explored in this lesson.

- When a linked expression will not fit on one line, division is made before the comparison sign that begins the link.
  - When the anchor will not fit on one line, division is made within the anchor.
  - When a link will not fit on one line, division is made within the link.
  - In a linked expression, if either the anchor or any link must be divided, a division must also be made before each link.
- Fractions are kept intact by dividing before an opening fraction indicator. If a fraction must be divided, division is made before the fraction line. A mixed number should not be divided.
- Transition to a runover line does not take the place of a necessary baseline indicator. The baseline indicator will be the first symbol on the new line.
- When a base and its superscript or subscript are too long to fit on a single braille line, division is made either within the base, within the superscript or subscript, or before a change-of-level indicator.
- A function name and its argument should not be divided at the space between them. If the entire expression will not fit on the line, division strategies are applied to the argument.
- Items within grouping signs should not be divided. If consecutive groupings do not fit on the line, a division may be made between groupings.
  - If a grouped expression will not fit on one line, division strategies are applied within the grouping. If the grouped expression is an enclosed list, division is made after a comma.
- Division may be made after a mathematical termination indicator such as termination of a radical expression or termination of a modified expression.
- When an embedded math expression must be divided, it may begin on the current line provided division is made in accordance with the principles defined in this section.
- When a displayed math expression must be divided, all runovers begin two cells to the right of the original display cell unless it is a nested linked expression in which case a second indent level is applied. (See Section 8.22 in Lesson 8.)

Submit Exercise 15 to your instructor.















