

LESSON 17

SYSTEM OF EQUATIONS

Enlarged Signs of Grouping

DETERMINANTS AND MATRICES

[Answers to Practice Material](#)

LESSON PREVIEW

Enlarged grouping signs are often encountered in the topics studied in this lesson: systems of equations and arrays (matrices and determinants). Format rules are given for these spatial arrangements, including further considerations regarding commentary.

SYSTEM OF EQUATIONS

[NC 25.9]

17.1 Definition and Recognition

A system of equations, sometimes called "simultaneous equations," is a collection of two or more equations which share variables. The student identifies the value of each variable by "solving the system." You can recognize a system by noticing the arrangement of equations on two or more lines. If the system consists of two equations, there will be two variables to solve (typically x and y); if the system consists of three equations, there will usually be three variables to solve (typically x , y , and z). The equations may or may not be joined by an enlarged grouping sign.

Here is a sample of a system of two equations using variables x and y .

$$\begin{array}{l} 4x - y = 10 \\ 2x = 12 - 3y \end{array}$$

17.2 Transcription Rules for Systems of Equations

A system of equations is a spatial arrangement and is transcribed as follows.

- One blank line is left above and below the system. Placement of code switch indicators follows the general rules for spatial arrangements.
- Use and nonuse of the numeric indicator follows Nemeth rules for nonspatial material, even though this is a spatial arrangement. (See also [subitem \(d\)](#), below.)
- Alignment is maintained only if terms and symbols are aligned in print. If terms are aligned on one side of the equals sign but not on the other side, follow print. Alignment is disregarded if only the equals signs are aligned.
- When a space is inserted within the equation for the purpose of maintaining alignment, a numeric indicator is not used.

The same rules apply to a system of inequalities. (See [Example 17-4](#)).

Note that, if a separation line is present, it is a spatially arranged addition problem and Nemeth rules for spatial addition are followed. (See Lesson 9.)

Example 17-1

18. Solve and check.

$$\begin{array}{l} 4x - y = 10 \\ 2x = 12 - 3y \end{array}$$

Print observation: This system has two variables (x , y). There are two equations. The terms are not vertically aligned.

PRACTICE 17A

1. Solve and check:

$$2x + 3y = 2$$

$$8x - 4z = 3$$

$$3y - 8z = -1$$

2. Solve:

$$2x - 5y + 6z = 11$$

$$3x - 2y + 3z = 9$$

$$2x + 4y - 9z = -3$$

3. Add:

$$3x - y = 7$$

$$2x + y = 8$$

$$\hline 5x + 0 = 15$$

Enlarged Signs of Grouping

[NC 19.6]

17.3 A Unified Expression

The Nemeth code calls an arrangement a "unified expression" when equations are grouped together using an enlarged grouping symbol. Here is a print sample of a unified system of equations, using a left enlarged brace.

$$\left\{ \begin{array}{l} x = y \\ 5x - y = 4 \end{array} \right.$$

17.4 Transcription Rules for Enlarged Signs of Grouping

- If only the left or right grouping sign is shown in print, only that symbol is shown in braille.
- Enlarged grouping symbols are transcribed on each line of the unified expression and are vertically aligned.
- When terms are not aligned in print, each line of the unified expression begins in the same cell. When there is a left enlarged grouping symbol, each expression will begin in the cell which immediately follows the left enlarged grouping symbol.

- (d) When the expressions require vertical alignment, at least one item must begin in the cell which immediately follows the left enlarged grouping symbol. The numeric indicator is not required before the first numeric character following a left grouping symbol.
- (e) At least one item ends in the cell which immediately precedes the right enlarged grouping symbol.

17.4.1 **Left Enlarged Brace.** The left enlarged brace curves and points to the left in print. Notice that the enlarged braille symbol is formed by inserting a dot 6 before the second cell of the normal brace symbol.

⠠⠨	Left Brace (normal size)	{
⠠⠠⠨	Left Enlarged Brace (covering two or more lines)	

Example 17-5

$$\left\{ \begin{array}{l} x = y \\ 5x - y = 4 \end{array} \right.$$

Print observation: A left enlarged brace groups the unified system of equations. Terms are not vertically aligned.

1	⠠⠨	
2	⠠⠠⠨	
3	⠠⠠⠨	⠠⠠⠨
4	⠠⠠⠨	⠠⠠⠨
5	⠠⠠⠨	
6	⠠⠨	

Braille observation: The enlarged grouping symbols are aligned. The terms are not aligned in print; each equation begins in the cell following the left enlarged brace. A numeric indicator is not needed for the numeral 5 because it is not preceded by a space. A numeric indicator is required for the numeral 4.

17.4.2 **Right Enlarged Brace.** The right enlarged brace curves and points to the right in print. Notice that the enlarged braille symbol is formed by inserting a dot 6 before the second cell of the normal brace symbol.

⠠⠨	Right Brace (normal size)	}
⠠⠠⠨	Right Enlarged Brace (covering two or more lines)	

Lines 1-2: The paragraph begins in cell 3. The runover is in cell 1.

Line 3: A blank precedes the spatial arrangement that is embedded in the paragraph.

Line 4: The narrative continues, in the runover cell of the paragraph. The opening switch is placed on this line only. The narrative continues after the top right enlarged grouping symbol.

Line 5: The second line of the embedded spatial arrangement is aligned with the line above, starting in the same cell and with the right grouping symbols aligned.

Line 6: A blank follows the spatial arrangement that lies within the paragraph.

Line 7: The paragraph continues.

PRACTICE 17B

After solving for x and y , we determine that the system $\begin{cases} 2x - 3y = 17 \\ 3x + 2y = 6 \end{cases}$ has the solution set $\{(4, -3)\}$. Name the solution set for this system.

$$\begin{cases} x + 2y = 6 \\ 2x - y = 7 \end{cases}$$

17.6 Enlarged Parentheses

Notice that the enlarged braille parentheses are formed by inserting a dot 6 before the normal parenthesis symbol.

⠠	Left Parenthesis (normal)	(
⠠⠠	Left Enlarged Parenthesis (covering two or more lines)	
⠡	Right Parenthesis (normal))
⠡⠡	Right Enlarged Parenthesis (covering two or more lines)	

17.7 Placement of Symbols

Symbols which appear outside of the enlarged grouping symbol, and which apply to the arrangement, are placed on the top line of the arrangement, even if the items are centered in print.

PRACTICE 17D

Find x and y in terms of a and b .

1.
$$\begin{cases} ax + by = 0 \\ x + y = 1 \end{cases} \quad (a \neq b)$$

2.
$$\begin{cases} ax + by = 0 \\ a^2x + b^2y = 1 \end{cases} \quad (a \neq 0, b \neq 0, a \neq b)$$

3.
$$\left. \begin{array}{l} a = \frac{x+y}{x-y} \\ b = \frac{x-y}{x+y} \end{array} \right\} -1 < x < 1, -1 < y < 1$$

4. Solve.

$$\left. \begin{array}{l} 11z - 4z + 2z = 27 \\ 7z + 2z = 27 \\ 9z = 27 \\ z = 3 \end{array} \right\} \text{Combine like terms.}$$

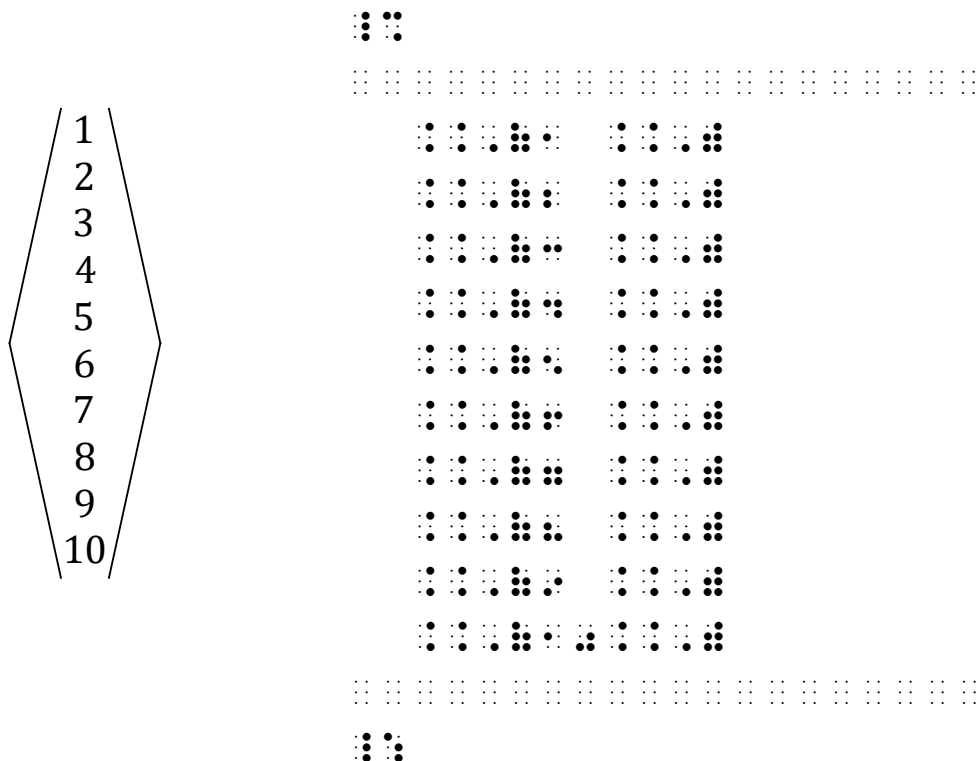
17.11 More Enlarged Signs of Grouping [NC Rule 19]

In addition to the enlarged braces and parentheses, the Nemeth code provides symbols for six other enlarged signs of grouping. Notice that each enlarged braille symbol is formed by inserting a dot 6 before the ⠠, ⠡, or ⠢ symbol of the normal-sized grouping symbol.

Vertical Bar		
⠠⠼	Single, Normal	
⠠⠠⠼	Double, Normal	
⠠⠠⠠⠼	Single, Enlarged	
⠠⠠⠠⠠⠼	Double, Enlarged	
Barred Brace		
⠠⠠⠠⠼	Left, Normal	{
⠠⠠⠠⠽	Right, Normal	}
⠠⠠⠠⠠⠼	Left, Enlarged	
⠠⠠⠠⠠⠽	Right, Enlarged	
Bracket		
⠠⠠⠼	Left, Normal	[
⠠⠠⠽	Right, Normal]
⠠⠠⠠⠼	Left, Enlarged	
⠠⠠⠠⠽	Right, Enlarged	
Angle Bracket		
⠠⠠⠠⠼	Left, Normal	<
⠠⠠⠠⠽	Right, Normal	>
⠠⠠⠠⠠⠼	Left, Enlarged	
⠠⠠⠠⠠⠽	Right, Enlarged	

Barred Bracket		
⠠⠠⠠	Left, Normal	⠠
⠠⠠⠠	Right, Normal	⠠
⠠⠠⠠⠠	Left, Enlarged	
⠠⠠⠠⠠	Right, Enlarged	
Half Bracket		
⠠⠠⠠	Upper Left, Normal	⠠
⠠⠠⠠	Upper Right, Normal	⠠
⠠⠠⠠⠠	Upper Left, Enlarged	
⠠⠠⠠⠠	Upper Right, Enlarged	
⠠⠠⠠	Lower Left, Normal	⠠
⠠⠠⠠	Lower Right, Normal	⠠
⠠⠠⠠⠠	Lower Left, Enlarged	
⠠⠠⠠⠠	Lower Right, Enlarged	

Example 17-17



PRACTICE 17E

Instructions: Follow "top alignment" rules for these side-by-side arrays. Place the leftmost character in cell 1.

$$\langle x \rangle \langle \begin{array}{c} x \\ y \\ z \end{array} \rangle \langle \begin{array}{c} u \\ v \\ w \\ x \\ y \\ z \end{array} \rangle$$

DETERMINANTS AND MATRICES
[NC 25.8]
17.12 Definition and Recognition

Determinants and matrices (singular: matrix) are arrangements of items in rows and columns which are enclosed between left and right grouping symbols. Items can be numbers, symbols, or mathematical expressions.

Here is a 2×2 ("two by two") determinant enclosed between enlarged vertical bars.

$$\left| \begin{array}{cc} a & b \\ c & d \end{array} \right|$$

Here is a 2×3 ("two by three") matrix enclosed between enlarged brackets.

$$\left[\begin{array}{ccc} 1 & 9 & -13 \\ 20 & 5 & -6 \end{array} \right]$$

A matrix can also be composed of only one column or only one row. Row matrices will be studied at the end of this section.

Determinants and matrices may also be referred to as "arrays."

17.13 Transcription Rules for Determinants and Matrices

Observe the following rules regarding the transcription of the two samples shown in [Section 17.12](#).

PRACTICE 17F

1. $\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

2. $\begin{pmatrix} 1 & -\frac{3}{4} & \frac{5}{3} \\ 2 & 5 & 12 \end{pmatrix}$

3. $\begin{bmatrix} a & b & c \\ 0 & 0 & 0 \end{bmatrix}$

4. $\begin{vmatrix} ab & cd \\ ac & ce \end{vmatrix}$

5. Explain why points
- (a_1, b_1)
- ,
- (a_2, b_2)
- , and
- (a_3, b_3)
- are collinear if and only if

$$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0$$

PRACTICE 17G

- A. The unit vectors of a three dimensional Cartesian coordinate system are shown below.

$$\hat{\mathbf{i}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \hat{\mathbf{j}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \text{ and } \hat{\mathbf{k}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

17.15 Ellipses, Single Dots, and Blank Entries

The following symbols represent ellipses, a single dot, or a blank entry in a matrix or a determinant.

$\cdot\cdot\cdot$	Blank Entry	
$\cdot\cdot\cdot\cdot$	Ellipsis, Diagonal, lower left to upper right	$\cdot\cdot$
$\cdot\cdot\cdot\cdot$	Ellipsis, Diagonal, upper left to lower right	$\cdot\cdot$
$\cdot\cdot\cdot\cdot$	Ellipsis, Horizontal	\cdots
$\cdot\cdot\cdot\cdot$	Ellipsis, Vertical	\vdots
\cdot	Single Dot	\cdot

Each symbol is positioned as far left as possible in its column. A transcriber's note must explain the use of the short dash to represent the blank space. Sample transcriber's note:

A short dash represents a blank entry.

Example 17-24

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

Print observation: A horizontal ellipsis is in each entry in column 3. A single dot is printed in row 3, columns 1, 2, and 4.

Lines 3-6: Note that this example uses full margins in addition to applying runover techniques.

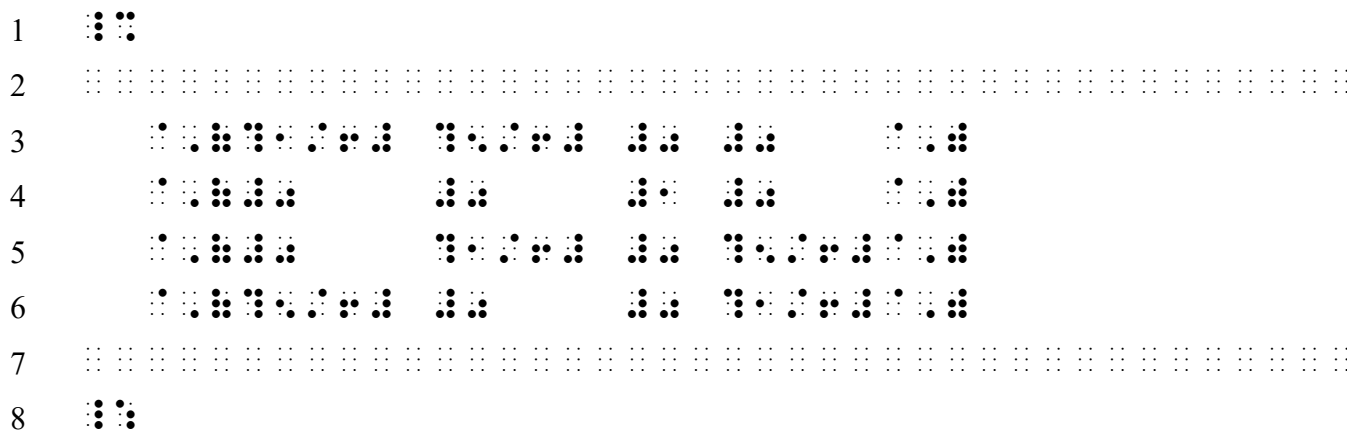
Lines 3 and 5: A baseline indicator is not needed between the superscript "t" and the right grouping symbol because the expression continues on the next line.

Lines 4 and 6: The runover line of each row entry ends with the letter t in the superscript position. A baseline indicator is required before transcribing the right grouping symbol which follows, unspaced.

17.17.4 **Fractions in Arrays.** Fractions in an array may be transcribed linearly but may be arranged spatially if linear fractions take up too much space.

Example 17-32

$$\begin{bmatrix} \frac{1}{6} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} \end{bmatrix}$$



Observe the following details when transcribing the fractions spatially.

- A line is skipped between the rows containing the spatial fraction. Enlarged grouping symbols are transcribed on those blank lines within the arrangement.
- Operation signs and variables are placed on the same line as the spatial fraction line.
- Each entry is moved as far up as possible in its row, including entries that are not fractions.
- If the last item in a row is a superscript, or a subscript that uses a subscript indicator, a return to the baseline must occur before the right grouping symbol is transcribed. If the row extends fully to the grouping symbol, a baseline indicator is required to return to the baseline. If the row does not extend to the right grouping symbol, the space returns the reader to the baseline.

PRACTICE 17J

Instructions: Use spatial fractions.

The derivative of T can be expressed as follows.

$$dT = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \dots & \frac{\partial y_1}{\partial x_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \dots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}$$

PRACTICE 17K

Instructions: Use runovers with indentation. Review spacing rules and "keep together" rules for abbreviated function names in Lesson 14.

In this Jacobian determinant,

$$\begin{vmatrix} 0 & 5 & 0 \\ 8x_1 & -2x_3 \cos(x_2x_3) & -2x_2 \cos(x_2x_3) \\ 0 & x_3 & x_2 \end{vmatrix} = -8x_1 \begin{vmatrix} 5 & 0 \\ x_3 & x_2 \end{vmatrix} = -40x_1x_2$$

the orientation of the resulting object is reversed.

PRACTICE 17M

1) Here are three examples of matrix operations.

a) $[5 \ 4] + [20 \ 30] = [25 \ 34]$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 1 & 5 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 7 \\ 1 & 9 \end{bmatrix}$

c) $[2 \ 1] \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = [-1 \ 2]$

17.20 Use of Tactile Graphics for Enlarged Grouping Signs

Enlarged grouping symbols may be drawn in place of the braille equivalents, especially when space saving is a factor. Refer to BANA's *Guidelines and Standards for Tactile Graphics* for drawing techniques. See Section 12.6.3 in Lesson 12 for horizontal grouping signs.

For further practice, see Addendum 1—Reading Practice.

Submit Exercise 17 to your instructor.
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PRACTICE 17D

1 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

2 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

3 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

4 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

5 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

6 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

7 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

8 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

9 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

10 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

11 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

12 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

13 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

14 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

15 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

16 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

17 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

18 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

19 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

20 $2x^2 + 3x + 4 = 5x^2 + 6x + 7$

Line 4: A space is inserted before x and y because terms are aligned, as printed.

Lines 6-7: Although the equals signs are aligned in print, terms are not aligned. In braille, each line is left adjusted.

Line 8: An additional transcriber-inserted grouping symbol is needed because the remark takes three lines.

Lines 15-18: Although this is not a system of equations (there is only one variable), alignment guidelines in [Section 17.2\(c\)](#) are followed. Terms are aligned on the left of the equals signs; to the right, place values are not aligned.

