

## LESSON 6

- [LEVEL INDICATORS](#)
  - [Superscripts](#)
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### *Format*

- [Itemized Material with Subdivisions](#)
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### LESSON PREVIEW

This lesson begins by looking at format. Itemized material with subdivisions is found throughout math textbooks, in exercise sets, and in answer sections. The rules differ somewhat from those followed in a nontechnical transcription. The topic of superscripts and subscripts is presented. Superscript, subscript, and baseline indicators are introduced. The lesson ends with another look at grouping signs as they relate to level indicators.











- b. **When Not To Retain Column Format.** If all the columns cannot be accommodated across the braille page, each subdivision in each problem must be lettered individually and the format in [Section 6.1](#) must be followed.

*Example 6-8*

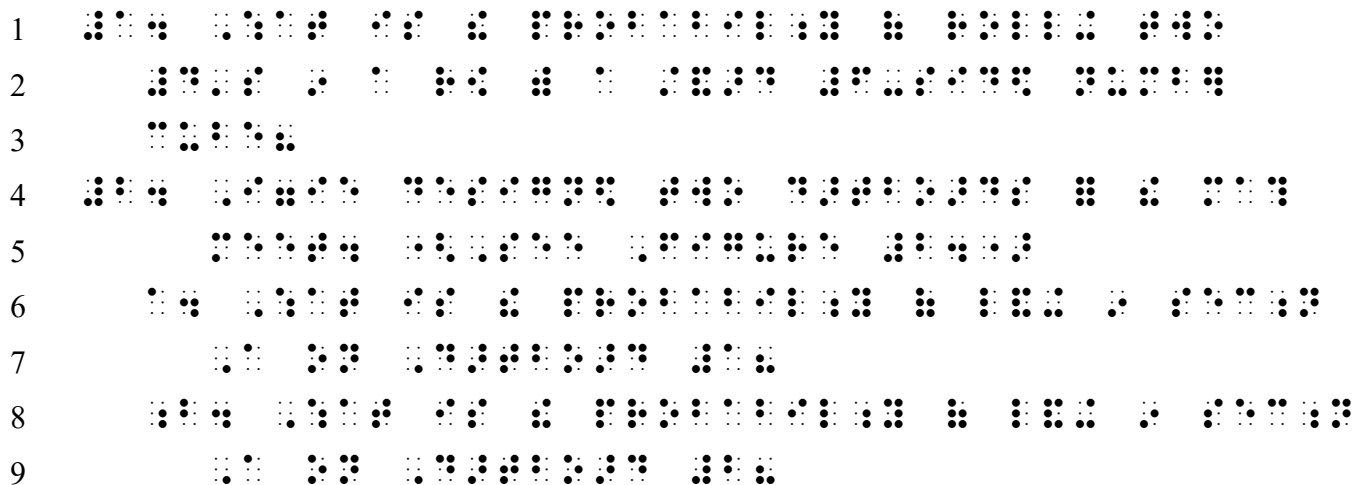
	a	b	c	d
1.	16 + 9	17 + 4	14 + 23	37 + 18
2.	46 + 15	87 + 12	95 + 54	101 + 43
3.	157 + 452	134 + 63	458 + 12	935 + 298

1	⠠⠠			
2	⠠⠠⠠⠠			
3	⠠⠠	⠠⠠⠠⠠		
4	⠠⠠	⠠⠠⠠⠠		
5	⠠⠠	⠠⠠⠠⠠⠠⠠		
6	⠠⠠	⠠⠠⠠⠠⠠⠠		
7	⠠⠠⠠⠠			
8	⠠⠠	⠠⠠⠠⠠⠠⠠		
9	⠠⠠	⠠⠠⠠⠠⠠⠠		
10	⠠⠠	⠠⠠⠠⠠⠠⠠		
11	⠠⠠	⠠⠠⠠⠠⠠⠠⠠⠠		
12	⠠⠠⠠⠠			
13	⠠⠠	⠠⠠⠠⠠⠠⠠⠠⠠		
14	⠠⠠	⠠⠠⠠⠠⠠⠠⠠		
15	⠠⠠	⠠⠠⠠⠠⠠⠠		
16	⠠⠠	⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠	

6.1.4 **Varied Margins.** Runover margins for itemized material are determined individually for each item. That is, an item with no subdivisions will be (1-3); the next item may have subdivisions and so will be (1-5; 3-5), etc.

*Example 6-9*

1. What is the probability of rolling two 4's in a row with a standard 6-sided number cube?
2. Iggie designed two dartboards for the math meet. (See Figure 2.)
  - a. What is the probability of landing in section A on Dartboard 1?
  - b. What is the probability of landing in section A on Dartboard 2?

















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**PRACTICE 6D**

- (1)  $4(x - y)^3 - 2(x - y)^3$
  - (2)  $3a^3b + 6a^6b^2 + 9a^9b^3$
  - (3) Convert 10° F to °C. Express 10 °C in °F.
  - (4)  $V = 2\pi^2 Rr^2$
  - (5)  $x^n$ -dimensional system
  - (6)  $6^2 \times 6^3 = 6^{2+3} = 6^5$
  - (7)  $2(36x^2 - 1) = 2[(6x)^2 - 1^2]$ , which factors into  $2(6x + 1)(6x - 1)$ .
-







## *Subscripts*

### 6.9 Subscript Level Indicators

Except as stated in [Section 6.11](#), the subscript level indicator is used to show that the symbols immediately following it appear on the first level below the baseline of writing. Note that the subscript indicator is the same symbol as the English-letter indicator—dots 56. The indicator's function is understood in context.

⠠ Subscript Indicator
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➤  $f_n$       ⠠⠠⠠

➤  $a_{(k+1)}$       ⠠⠠⠠⠠⠠⠠⠠

Subscripts may carry subscripts of their own. In such cases, the subscript level indicator is doubled, tripled, etc. to indicate subscripts on the second, third, or lower levels.

⠠⠠ Subscript with Subscript (two levels below the baseline)
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⠠⠠⠠ Subscript with Subscript with Subscript (three levels below the baseline)
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➤  $n_{x_y}$       ⠠⠠⠠⠠⠠⠠

➤  $P_{x+1y+1z+1}$       ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

### PRACTICE 6F

*Note:* The -?- in the second column is in the subscript position. The subscript of the second item in the fourth column is "minus two".

$3_c$        $10_8$        $?_3$        $x_{2+k}$

$3_{\text{five}}$        $10_{-?}$        $\text{Ca}(\text{OH})_2$        $y_{-2}$

$y_{n_k}$        $P_{3n}$        $a_{m1}$        $a_{m_1}$









- b. A subscript level indicator must be used in the following circumstances.
- A numeral on the first level below the baseline requires a subscript indicator if the subscript contains any symbol other than a numeral with its comma or decimal point.
  - A numeral on the first level below the baseline requires a subscript indicator if the subscript carries a superscript or subscript of its own.
  - A subscript on the second or lower level always requires the appropriate subscript level indicator.
  - If the subscript is a letter that is part of a word or an abbreviation, a subscript indicator is required.
  - If the subscript is a letter functioning as a numeral in a nondecimal numeration system, a subscript indicator is required.

### PRACTICE 6H

- 1) These expressions need subscript indicators:  $y_{-2}$ ,  $x_{2+k}$ ,  $a_{m1}$ ,  $x_{3n}$ ,  $x_{y_2}$ .
- 2) These expressions do not need a subscript indicator:  $x_1$ ,  $ax_2$ ,  $\text{CO}_2$ ,  $z_{.4}$ ,  $\beta_2$ .
- 3) Decide whether these expressions require a subscript indicator and transcribe them correctly:  $\text{shape}_4$ ,  $Q'_2$ ,  $\text{C}_6\text{Fe}_2\text{O}_{12}$ ,  $n_k$ ,  $x_{2k}$ ,  $P_{r_{st}}$ ,  $D_{56}$ ,  $G_{9,999}$ , and the hexadecimal number  $2\text{E}6\text{B}_{16}$ .
- 4) **Chemistry.** While  $\text{Na}_2\text{ZnCl}_4$  could be cooled in the normal way,  $\text{Na}_2[\text{CoCl}_4]$  had to be quenched in the liquid  $\text{N}_2$ .
- 5)  $f_1(x) = g(x) \cdot q_2(x) + f_2(x)$





6.12.3 **Space Before and After a Comparison Sign.** The space following a superscript or a subscript returns the reader to the baseline.

$$\gg a^2 = a \cdot a$$

*The superscript is 2. The equals sign is on the baseline.*

Hence, when a comparison sign is within a superscript or a subscript, the level must be reinstated before the comparison sign. The indicator is unspaced from the comparison sign. However, the space after the comparison symbol preserves the level that is already in effect.

$$\gg S_{u=a}$$

*The subscript is  $u = a$ . The subscript level is restated before the equals sign. The subscript level continues through the space following the equals sign without the need for an explicit indicator.*

6.12.4 **Space Before and After an Ellipsis or Long Dash.** Established spacing rules for an ellipsis and a long dash apply when these symbols occur at the superscript or subscript level. Within a superscript or a subscript, the space which *precedes* an ellipsis or a long dash preserves the effect of a level indicator currently in force. Similarly, the space which *follows* an ellipsis or a long dash preserves the effect of any level indicator currently in force. In other words, within a superscript or a subscript the space does not initiate the baseline level.

$$\gg x^{1 + 3 + 5 + \cdots + (2n-1)}$$

$$x^{1 + 3 + 5 + \cdots + (2n-1)}$$

*This superscript includes an ellipsis. The level in effect preceding the ellipsis is a first order superscript. That level extends through the spaces before and after the ellipsis. (See Section 2.16.3 in Lesson 2 regarding the midline ellipsis.)*

a. **A Change of Level Before or After an Ellipsis or Long Dash.** When a change of level occurs before or after an ellipsis or long dash, the following rules must be applied in order to offset the fact that such a space does not bring about a level change.

(1) When an ellipsis or a long dash is located at a different level from the material *preceding* it, the appropriate level indicator is used before the ellipsis or long dash. The level indicator is unspaced from the ellipsis or long dash. Compare the next two samples.

$$\gg r_n \dots$$

*This ellipsis is in the subscript. The subscript indicator applied to the letter  $n$  continues through the space.*









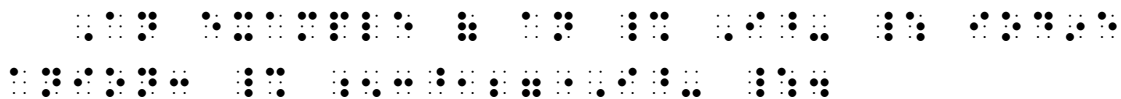






**Example 6-42**

An example of an I<sup>-</sup> iodine anion:  ${}^{127}_{53}\text{I}^-$ .



The subscript indicator is or is not transcribed, according to the rules presented in [Section 6.11](#). Notice that simultaneous superscripts and subscripts printed next to a letter in italics may not align exactly because of the slanted aspect of the italic typeform.

$$\gg x_1^2 \quad \text{⠠⠭⠨⠠⠆⠨⠠⠆}$$

*The numeric subscript does not require a subscript indicator because it is a right subscript to a letter.*

$$\gg {}_m^n x \quad \text{⠠⠆⠠⠨⠠⠭⠨⠠⠆}$$

*The subscript is transcribed first.*

$$\gg n_{abc}^{xyz} \quad \text{⠠⠨⠠⠆⠠⠃⠠⠃⠠⠭⠠⠗⠠⠗⠠⠆⠠⠃⠠⠃⠠⠆}$$

*The entire subscript is transcribed before transcribing the superscript.*

## 6.18 Nonsimultaneous Superscripts and Subscripts

When a mathematical expression carries both a superscript and a subscript which are not printed directly above and below each other, the superscript and subscript are transcribed in the same order as in print, and the baseline indicator is inserted between them.

$$\gg x_1^2 \quad \text{⠠⠭⠨⠠⠆⠨⠠⠆}$$

$$\gg a_m^k \quad \text{⠠⠁⠨⠠⠆⠨⠠⠆}$$

$$\gg y_x^n \quad \text{⠠⠭⠨⠠⠆⠨⠠⠆}$$

$$\gg b_a^n \quad \text{⠠⠃⠨⠠⠆⠨⠠⠆}$$

A magnifier and a straightedge can help determine whether superscripts or subscripts are simultaneous or nonsimultaneous. If in doubt whether the expression shows nonsimultaneous super/subscripts or if, instead, the super/subscripts have super/subscripts of their own, compare the notation to the surrounding text for clues.



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**PRACTICE 6K**

*Note:* There is no space between the subscript and the superscript in item #3.

1. Here are some expressions with left superscripts:  ${}^3x$ ,  ${}^nx$ ,  
 ${}^{-2} + {}^{-4} = {}^{-6}$ ,  $(-3)^{{}^{-2}+{}^{+2}}$ .
  2.  ${}^{12}_6\text{C}$  and  ${}^{12}\text{C}$  represent the same carbon isotope.
  3.  $\text{D}_2{}^{18}\text{O}$  is the doubly labeled water isotopologue!
  4. In  $\text{CO}_2$ , the subscript  $_2$  means "two oxygen atoms".
  5.  ${}_nP_r = K({}_{n-1}P_{r-1})$
  6.  $a_1^2 + b_1^2 + c_1^2$
  7.  $[t]_0^4$
  8.  $2 \times 10_6^2 + 3 \times 10_6^1 + 2$
  9.  $P_{xy}Q$
  10.  $\text{NH}_4^+ + \text{Cl}^- + \text{H}_2\text{O}$
-

### 6.21 More About Grouping Symbols and Level Indicators

As noted throughout this lesson, when a grouping symbol appears on the baseline level and a level indicator is currently in effect, the baseline indicator is placed before the sign of grouping.

$$\gg (R_H T_H) + (R_S T_S)$$



If no subscript indicator is used, a return to the baseline is implied.

$$\gg (a_1, a_2, a_3)$$

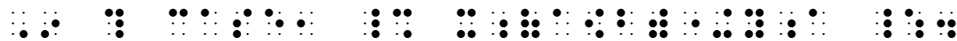
*This right parenthesis does not need a baseline indicator.*

Be watchful when determining the level of a grouping sign. Because the top and bottom margins of grouping signs may extend beyond the level in effect, it is better to compare the center of the symbol to the surrounding text if its printed level is in question.

- a. When a grouping sign starts at a different level, the appropriate level indicator is placed before the grouping symbol to properly identify its location. Review the examples in [Section 6.7.1](#) with this in mind.

Example 6-45

In this case,  $x_{(a,b)} + y_a$ .

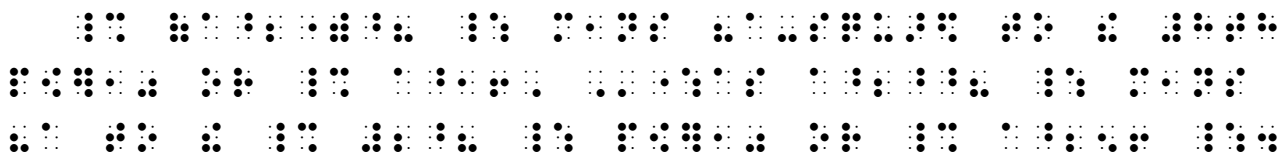


*The left grouping symbol begins the subscript level.*

- b. When a grouping symbol is printed on the baseline of writing, an associated exponent is at the superscript level, even though it may be raised to a higher level in print. This is a visual aid, not to be mistaken for a higher-degree exponent.

Example 6-46

$(a^2)^8$  means "a-squared to the 8<sup>th</sup> power," or  $a^{16}$ , whereas  $a^{2^8}$  means "a to the 2<sup>8</sup> power," or  $a^{256}$ .



*The first exponent 8 is a superscript to  $(a^2)$ . The second exponent 8 is a supersuperscript.*

- c. Lesson 4 explained that, when a letter touches only one grouping symbol, the English-letter indicator is applied (or is not applied) as though the grouping sign were not present. This rule is illustrated below in the context of a grouping sign that has a simultaneous subscript and superscript.

⊳  $s]_a^b$       ⋮ ⋮ ⋮ ⋮ ⋮ ⋮

Think:  $s_a^b$       ⋮ ⋮ ⋮ ⋮ ⋮

### PRACTICE 6L

*Instructions:* Treat each vertical bar in sentence (6) as an operation sign.

- (1)  $\{f_n\}$
- (2)  $|a_m - a_n|$
- (3)  $(x_1y_1 + x_2y_2)$
- (4)  $([\text{CH}_3]_2\text{CH})$
- (5)  $I_{\text{ue}}^{2''} = (\text{H}'_{44}x'_{\text{ve}})^{+'}$
- (6) The dagger and the asterisk are used as superscripts in quantum mechanics:  $A^\dagger, (x^\dagger)^\dagger = x, \langle \phi | \psi \rangle^* = \langle \psi | \phi \rangle$ .

*For further practice, see Addendum 1—Reading Practice.*

Submit Exercise 6 to your instructor.



PRACTICE 6C

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10
- 11
- 12
- 13
- 14
- 15

Runover margins for itemized material are determined individually for each question. Item 2 has subdivisions—its runover position (line 5) is cell 5.







## PRACTICE 6I

1.  $x_{1,2} \neq x^{i,j}$
2.  $x_{n-1,n-1}, x_{n-1,n}, x_{n,n-1}$
3.  $A^{n+n+n}$  all  $n$ 's are equal.
4.  $s]_{t=a}$
5.  $e^{1,000}$
6.  $a^{m+k} \div a^m = a^k$
7.  $P_{s_1 \dots s_2}$  and  $P_{q,r,s}$
8.  $10_{-?-} = 6_8$
9.  $a'_1, a'_2, \dots, a'_n$  are the inverses.
10.  $\aleph_0$  represents the cardinality of the set of all natural numbers.



