LESSON 10

LESSON PREVIEW

The study of spatial arrangements continues with a look at multiplication and division problems. The lesson ends with a review of the rules for use/nonuse of the numeric indicator and a summary of rules regarding signs of grouping.

NOTE: Code switch indicators are not shown in the isolated Nemeth examples in this lesson.

SPATIAL ARRANGEMENTS, continued

Lesson 9 looked at spatially arranged addition and subtraction problems. The list below summarizes format rules presented in that lesson.

Review of Format for Spatial Arrangements

1. A blank line is required above and below a spatial arrangement. It may begin on line 1 and/or end on line 25.
2. Code switch indicators are placed outside of the arrangement.
3. The numeric indicator is not used. (There are some exceptions.)
4. An identifier associated with a spatial arrangement is positioned according to rules applied to the specific type of problem.
5. Side-by-side arrangement is allowed according to certain spacing rules.
6. If a spatial arrangement lies on lines 1-2, any symbol of the arrangement cannot appear within three blank spaces of the first symbol of the print page number on line 1. If a spatial arrangement lies on lines 24-25, any symbol of the arrangement cannot appear within three blank spaces of the first symbol of the braille page number on line 25.
7. Only the general omission symbol is used to show an omission within a spatial arrangement.

SPATIAL ARRANGEMENT WITH MULTIPLICATION

[NC 25.4]

The parts of a spatial multiplication problem are labeled in this illustration.

2	*multiplicand*
× 3	*multiplier*
——	*separation line*
6	*product*

10.1 Alignment

In a spatial arrangement for multiplication, the numerals, decimal points, and/or commas in the multiplier and multiplicand must be aligned the same way as in the print copy. An associated monetary symbol must be unspaced from the numeral to which it applies.

10.2 Placement of Multiplication Symbol

In braille, the multiplication symbol must immediately precede the multiplier, regardless of print placement.

A multiplication sign is not always present. First, examine the problem and the surrounding text to determine that this is indeed a multiplication problem. Then apply alignment rules for multiplication.

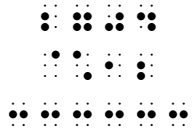
10.3 Separation Line

In braille, the separation line extends one cell to the left and to the right of the longest entry appearing above or below it. If there is more than one separation line in a given arrangement, each must be the same length regardless of the way it is printed.

⠶⠶⠶⠶⠶⠶	Separation Line (varying in length)

Example 10-1

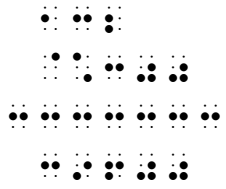
2704
× 12



Alignment: Digits in the multiplier and multiplicand (lines 1 and 2) are vertically aligned the same way as in the print copy. Placement of Multiplication Symbol: The multiplication cross is unspaced from the multiplier (line 2) even though this symbol is printed further to the left. Separation Line: The separation line extends one cell to the left and to the right of the longest entry appearing above or below it.

Example 10-2

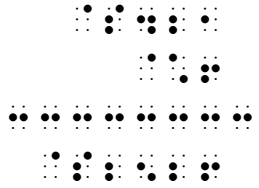
132
× 300
39600



Alignment: Digits in the multiplier and multiplicand (lines 1 and 2) and in the product (line 4) are vertically aligned the same way as in the print copy. Placement of Multiplication Symbol: The multiplication cross is unspaced from the multiplier (line 2) even though this symbol is printed further to the left. Separation Line: The separation line extends one cell to the left and to the right of the longest entry appearing above or below it.

Example 10-3

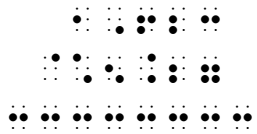
$421
× 6
$2526



Alignment and Spacing: Numerals are aligned as printed. Each dollar sign is unspaced from the following numeral. In braille, the multiplication cross is unspaced from the multiplier.

Example 10-4

$$\begin{array}{r} 1{,}623 \\ \times\ 5.27 \\ \hline \end{array}$$



Spacing and Alignment: The commas and decimal points correspond to the print placement.

Alignment of Partial Products

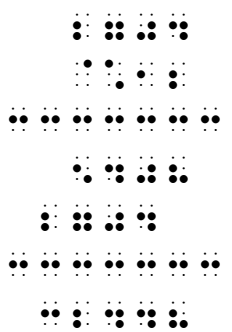
10.4 Partial Products

When partial products are shown in a sample problem, note that the final answer (the product) is obtained by adding the partial products. Hence, partial products and the final product (the answer) must be aligned for addition. All of the separation lines in one multiplication problem are the same width and in the same cell column, even though they may not appear this way in print.

2704	*multiplicand*	
× 12	*multiplier*	
5408	*partial product*	⎫
2704	*partial product*	⎬ *an addition problem*
32448	*product*	⎭

Example 10-5

$$\begin{array}{r} 2704 \\ \times\ 12 \\ \hline 5408 \\ 2704 \\ \hline 32448 \end{array}$$



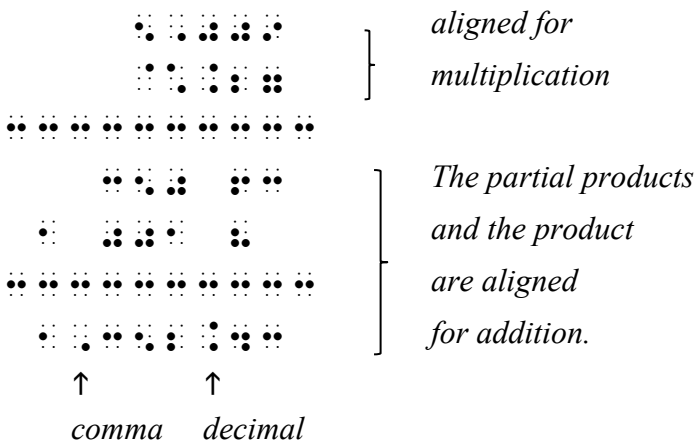
The multiplicand and the multiplier (lines 1 and 2) are aligned for multiplication, as printed.

The partial products (lines 4 and 5) and the product (line 7) are aligned according to the rules for addition, that is, by place value.

10.4.1 **Spacing.** If the product contains a comma or a decimal point, the corresponding cells in the partial products above it are left blank. Blank cells are not inserted in the separation lines.

Example 10-6

```
  5,009
× .27
 35063
 10018
1,352.43
```



PRACTICE 10A

Instructions: Use side-by-side layout, leaving one blank space between the end of one separation line and the beginning of the next. Include switch indicators in your transcription. Review side-by-side format rules in Lesson 9.

```
  $98        $33        12.12        7,165
× 100      × 200      × 15.3       × 85
```

```
  $98        $33        12.12        7,165
× 100      × 200      × 15.3       × 85
$9,800     $6,600      3636         35825
                      6060        57320
                      1212        609,025
                     185.436
```

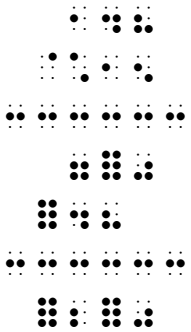
10.5 Omissions in Spatial Multiplication Problems

As noted in Lesson 9, only the general omission symbol is used to show an omission within a spatial arrangement, regardless of the print sign used. The one-cell symbol allows the transcription to maintain alignment.

⠨ General Omission Symbol

Example 10-7

$$\begin{array}{r} 148 \\ \times\ 15 \\ \hline 7\square 0 \\ \square 48 \\ \hline \square 2\square 0 \end{array}$$

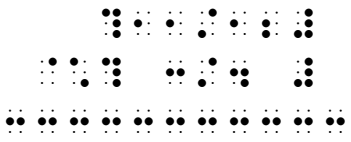


10.6 Fractions and Mixed Numbers

In a multiplication problem which contains fractions and mixed numbers, the various parts of the fractions are aligned vertically, both above and below the separation line, as in an addition or subtraction problem. You may wish to review Section 9.22 in Lesson 9.

Example 10-8

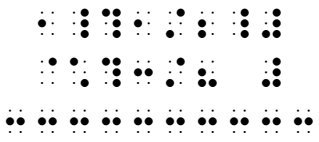
$$\begin{array}{r} \frac{11}{12} \\ \times\ \frac{3}{4} \\ \hline \end{array}$$



The fraction indicators and the fraction lines are vertically aligned. Numerators and denominators touch the fraction line.

Example 10-9

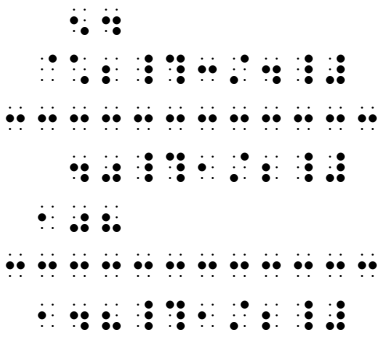
$$\begin{array}{r} 1\frac{1}{2} \\ \times\ \frac{3}{8} \\ \hline \end{array}$$



The fraction indicators align, including the fractional part of the mixed number. The multiplication cross touches the multiplier according to rules regarding spatial multiplication.

Example 10-10

$$\begin{array}{r} 54 \\ \times\ 2\frac{3}{4} \\ \hline 40\frac{1}{2} \\ 108 \\ \hline 148\ \frac{1}{2} \end{array}$$



Place values and indicators are aligned throughout.

10.7 Polynomials

In a multiplication problem which contains polynomials, the multiplicand and multiplier (above the first separation line) are aligned as in the print copy. The partial products and final product are aligned following rules for addition.

Example 10-11

$$\begin{array}{r} 8r + 9s \\ 5r - 6s \\ \hline 40r^2 + 45rs \\ \quad - 48rs - 54s^2 \\ \hline 40r^2 - 3rs - 54s^2 \end{array}$$

1 

2 

3 

4 

5 

6 

7 

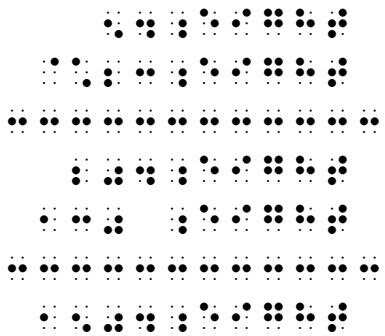
Lines 1-2: The multiplicand and multiplier are aligned as printed.
Lines 4-7: The partial products and the product are aligned for addition.
Lines 4-7: Terms, operation signs, superscript indicators, and baseline indicators are aligned.

10.8 Subscripts Denoting Nondecimal Bases

In arrangements which show multiplication of nondecimal bases, the subscript indicators are vertically aligned. The rightmost partial product sets the location of this alignment. This may differ from the spacing shown in the print copy.

Example 10-12

54_{eight}
$\times\ 23_{eight}$
204_{eight}
130_{eight}
1504_{eight}



To ensure that the subscript indicators align, a space is inserted in the second partial product.

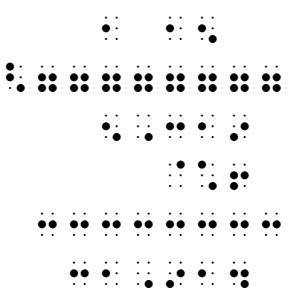
10.9 Regrouping Numbers with Multiplication

⠸⠿⠿⠿⠿⠿⠿	Regrouping Indicator for Numbers Above the Arrangement (varying in length)
⠰⠿⠿⠿⠿⠿⠿	Regrouping Indicator for Numbers Below the Arrangement (varying in length)

If regrouping numbers are shown, follow the rules for regrouping numbers with addition as outlined in Lesson 9. This indicator is transcribed whether or not the line appears in the print copy. It is inserted between the regrouping numbers and the multiplicand and is one cell wider than the separation line. The regrouping number is placed in the same columnar position as in print.

Example 10-13

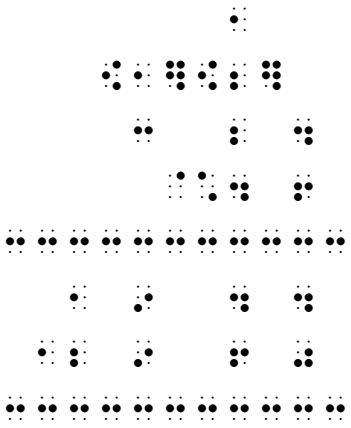
$^{1\ 1\ 5}$
$5{,}319$
$\times\ 6$
$31{,}914$



a. When the multiplier consists of more than one digit, the regrouping numbers might be canceled and replaced. Regrouping indicators are not needed when numbers are canceled. (Review Section 9.25 in Lesson 9.)

Example 10-14

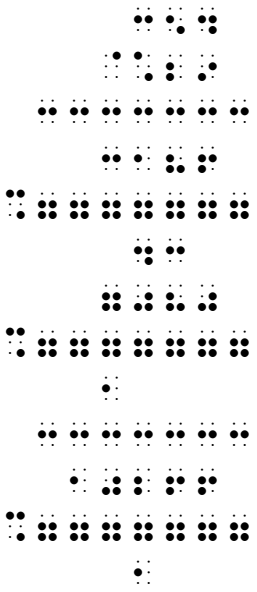
$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ \ } \\ 324 \\ \times\ 46 \\ \hline 1944 \\ 12960 \end{array}$$



b. When the multiplier consists of more than one digit, the regrouping numbers might be written within the partial products and product area. Determine whether the regrouping numbers are written above or below each digit to select the appropriate regrouping indicator – "above" or "below."

Example 10-15

$$\begin{array}{r} 354 \\ \times\ 29 \\ \hline 3186 \\ {\scriptstyle 4\ 3\ \ \ } \\ 7080 \\ {\scriptstyle 1\ \ \ \ } \\ \hline 10266 \\ {\scriptstyle \ \ 1\ \ \ } \end{array}$$



The appropriate regrouping indicator is inserted between the regrouping numbers and the line in the arrangement to which they apply.

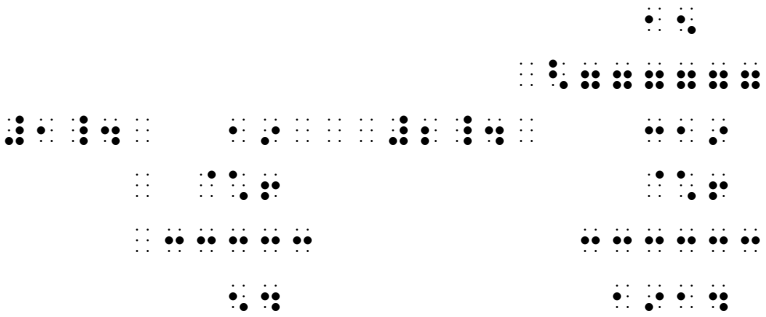
10.10 Placement of Identifiers with Spatial Multiplication

An identifier, if present, is placed on the first line of the multiplication problem (the multiplicand) regardless of its placement in print. If there are regrouping numbers, the identifier is still placed on the line with the multiplicand.

Example 10-16

1. $\begin{array}{r} 19 \\ \times\ 6 \\ \hline 54 \end{array}$

2. $\begin{array}{r} {\scriptstyle 1\ 5\ \ } \\ 319 \\ \times\ 6 \\ \hline 1914 \end{array}$



Reminders: One blank space comes between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines. No symbol of one spatial arrangement or its identifier may be less than three cells distant from any symbol on any line of a neighboring arrangement or its identifier, except at the ends of separation lines.

PRACTICE 10B

Instructions: Use side-by-side layout. Include code switch indicators in your transcription.

1) $\begin{array}{r} \frac{3}{4} \\ \times \frac{5}{8} \\ \hline \frac{15}{32} \end{array}$

2) $\begin{array}{r} 9\frac{3}{4} \\ \times 4\frac{7}{12} \\ \hline \end{array}$

3) $\begin{array}{r} {}^{3}\,{}^{3} \\ 999 \\ \times\ 4 \\ \hline 3{,}996 \end{array}$

4) $\begin{array}{r} 3p+6q \\ 11p-2q \\ \hline 33p^2+\ 66pq \\ -\ \ 6pq-12q^2 \\ \hline 33p^2+60pq-12q^2 \end{array}$

DIVISION PROBLEMS

[NC 25.5]

10.11 Notation Devices

It will be helpful to refer to the parts by using the following standard terminology. In this sample ("forty divided by ten equals 4"), 40 is the *dividend*, 10 is the *divisor*, and 4 is the *quotient*.

$$\begin{array}{r} \textit{quotient} \\ \textit{divisor}\overline{)\textit{dividend}} \end{array} \qquad \begin{array}{r} 4 \\ 10\overline{)40} \end{array}$$

Other print styles which can be found in various publications are illustrated below using the same numbers for dividend, divisor, and quotient. Each style includes a division sign (either straight, slanted, or curved) and a separation line. The quotient can be printed above, below, or next to the dividend. Any one publication will likely use only one style throughout a document.

$$10/\overline{40} \qquad 10\overline{)40}(4 \qquad 10\,\big|\,\overline{\begin{array}{r}4\\40\end{array}} \qquad \begin{array}{l} 10\,|\,40 \\ \overline{\quad 4} \end{array} \qquad \underline{10}\,\big|\,\overline{\begin{array}{r}4\\40\end{array}}$$

If there is a *remainder,* it may be shown after the quotient, preceded by the letter R or r. In this sample ("forty-seven divided by ten"), 7 is the remainder.

$$\begin{array}{r} \textit{quotient}\ \ \textit{remainder} \\ \textit{divisor}\overline{)\textit{dividend}} \end{array} \qquad \begin{array}{r} 4\ \text{R7} \\ 10\overline{)47} \end{array}$$

10.12 Linear (Nonspatial) Representation of Division Problems

The Preliminary Lesson introduced linear division problems using the symbol ÷ for "divided by", as in 15 ÷ 3 ("fifteen divided by three"). A division problem may also be printed using a curved, straight, or slanted line for the division sign, and a horizontal line printed above or below the dividend.

$$3\overline{)15}$$

When only a divisor and dividend are printed and there are no spaces within the dividend, the problem is formatted as a linear construction. The print style is replicated by using one of the following symbols between the divisor and dividend. Note that the straight and slanted division signs are represented with the same braille symbol.

⠸⠌	Curved Division Sign (curving right)	)
⠸⠡	Curved Division Sign (curving left)	(
⠸⠳	Straight Division Sign	|
⠸⠳	Slanted Division Sign (right or left)	/ or \

The vertical and slanted division symbol is formed with the same dot configuration as the punctuation indicator (dots 456). In context, it is clear that it is a vertical line, not a punctuation indicator. It is not necessary to explain this in a transcriber's note.

In linear format, the horizontal line printed above or below the dividend is not transcribed. If a horizontal line is printed below the divisor, it, too, is disregarded. The numeric indicator is used where required according to rules governing linear expressions. As with other operation signs, the division symbol is unspaced from the divisor and the dividend, even if the print copy shows a space.

⋙ $3\overline{)15}$ ⠼⠉⠸⠌⠼⠁⠑

⋙ $2\,|\overline{46}$ ⠼⠃⠸⠳⠼⠙⠋

⋙ $3\,\overline{/15.3}$ ⠼⠉⠸⠳⠼⠁⠑⠲⠼⠉

⋙ $\overline{1,000}\,(10$ ⠸⠳⠼⠁⠂⠚⠚⠚⠸⠡⠼⠁⠚

⋙ $\underline{30}|\overline{90}$ ⠼⠉⠚⠸⠳⠼⠊⠚

The horizontal line printed below this divisor is disregarded.

⋙ $40\overline{)\$49.99}$ ⠼⠙⠚⠸⠌⠫⠼⠙⠊⠲⠼⠊⠊

An unspaced symbol in the dividend does not negate this rule.

Example 10-17

Show how to solve $9\overline{)216}$ on your calculator. Is there a remainder?

⠠⠓⠕⠺ ⠞⠕ ⠎⠕⠇⠧⠑ ⠼⠊⠸⠌⠼⠃⠁⠋ ⠕⠝ ⠽⠕⠥⠗ ⠉⠁⠇⠉⠥⠇⠁⠞⠕⠗ ⠊⠎ ⠞⠓⠑⠗⠑ ⠁ ⠗⠑⠍⠁⠊⠝⠙⠑⠗⠲

Example 10-18

1. $3\overline{)15}$ 2. $0.3\overline{)30}$ 3. $3\overline{)4.5}$ 4. $30\overline{)9{,}060}$

1 ⠼⠉⠑

2 ⠼⠉⠲⠼⠚⠩ ⠼⠉⠲⠼⠙⠲⠑

3 ⠼⠉⠲⠼⠚⠩ ⠼⠉⠩⠼⠚⠲⠼⠚⠩

4 ⠼⠉⠲⠼⠑⠩ ⠼⠉⠲⠼⠙⠲⠼⠑

5 ⠼⠉⠲⠼⠑⠩ ⠼⠉⠲⠼⠙⠲⠑⠩⠼⠚⠲⠼⠚⠩ ⠼⠉⠲⠼⠁

Reminder: In nonspatial braille, each identifier begins on a new line in cell 1 even though the problems are arranged side by side in print.

PRACTICE 10C

Instructions: Use appropriate margins for embedded, displayed, and itemized problems.

Converting Fractions to Decimals

To write the fraction 1/2 as a decimal, you can divide "one point zero divided by two", or $2\overline{)1.0}$. Put away your calculator! Using mental math, it's easy! The answer is .5 .

Can you divide *these* fractions in your head?

1/5: $5\overline{)1.0}$

4/5: $5\overline{)4.0}$

9/10: $10\overline{)9.0}$

Practice

1. Solve these division problems.

   a. $8\overline{)128}$ b. $6\overline{)5{,}304}$

   c. $4\overline{)204}$ d. $9\overline{)837}$

10.13 Spatial Representation of Division Problems

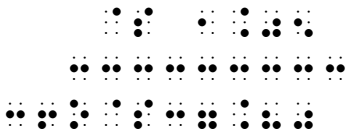
A division problem is transcribed as a spatial (vertical) arrangement if it includes a quotient, partial products and differences, or if there are spaces within the dividend. The quotient is aligned with the dividend as shown in the print copy. A monetary symbol follows print placement. See Example 10-19.

```
⠒⠒ ⠒⠒ ⠒⠒ ⠒⠒ ⠒⠒    Separation Line          ———
                    (varying in width)
```

10.13.1 **Width of the Separation Line.** The separation line begins in the column containing a division symbol and ends in the column containing the other division symbol if the latter appears in print. When only one division symbol is printed, the separation line ends in the cell which is one column beyond the overall arrangement. *Reminder:* The division symbol is unspaced from the divisor and the dividend.

Example 10-19

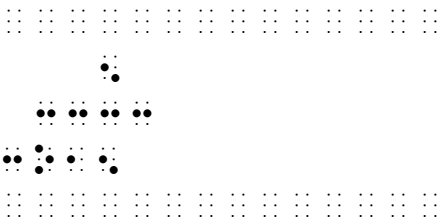
```
   $ 1.05
36)$37.80
```



A space follows the dollar sign in the quotient because monetary symbols follow print placement.

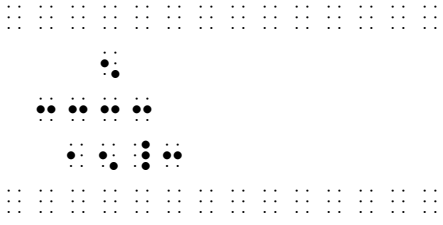
Several styles of division problems are illustrated below.

```
    5
3)15
```



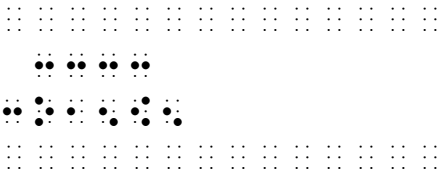
This separation line begins in the cell directly over the (curved) division sign and ends one cell beyond the overall arrangement.

```
 5
15| 3
```



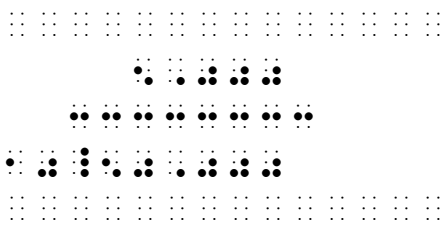
This separation line begins one cell beyond the overall arrangement and ends in the cell directly over the (straight) division sign.

⠸⠸ $3\overline{)15}(5$



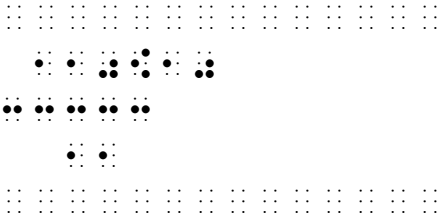
This separation line begins directly over the "curving right" division sign and ends in the cell directly over the "curving left" division sign. Because a quotient is present, it is transcribed as a spatial arrangement, even though the quotient is printed on the same line as the divisor and the dividend.

⠸⠸ $10\overline{)50{,}000}\ 5{,}000$



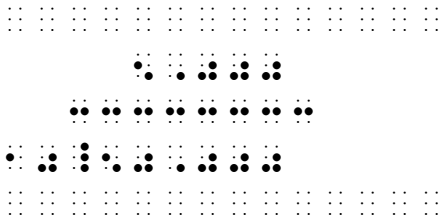
The quotient is aligned with the dividend as shown in the print copy.

⠸⠸ $\frac{110}{11}(10$



This arrangement shows the separation line printed below the dividend, with the quotient printed below that. The separation line begins one cell beyond the overall arrangement and ends in the cell directly under the division sign.

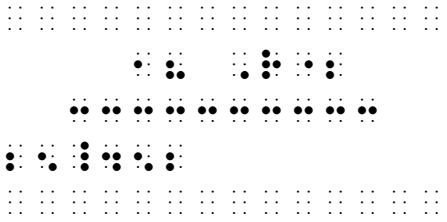
⠸⠸ $\underline{10}\overline{)50{,}000}\ 5{,}000$



This arrangement shows a horizontal line printed below the divisor. That line is disregarded in braille.

10.13.2 **A Quotient with a Remainder.** When a remainder is printed next to the quotient, the capitalized letter R, or uncapitalized letter r, (indicating "remainder") is preceded by a space. When the quotient is arranged above the separation line, the separation line extends one cell to the right of the overall arrangement, including the remainder, even if the line is not printed in this manner.

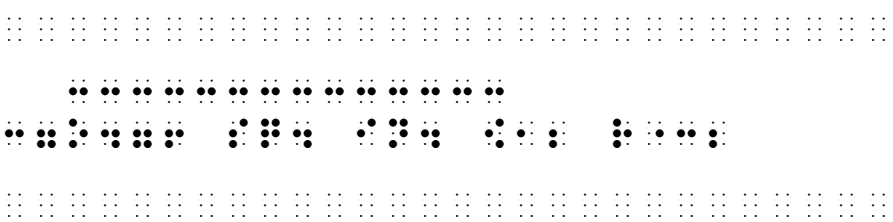
⋙ $25\overline{)452}$ 18 R2



A number printed to the right of a letter needs a multipurpose indicator (dot 5) to show that the number is not a subscript. (Lesson 6)

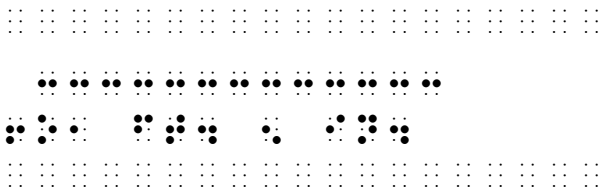
The arrangement in braille reflects the print layout. In the next sample, the quotient and remainder are printed to the right of the dividend.

⋙ $37\overline{)476}$ sq. in. (12 r 32



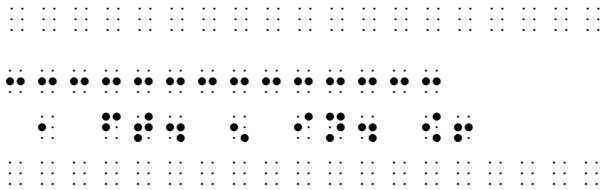
10.13.3 **Spaces and Spacing.** When there is a space within the divided a spatial arrangement is required. When an abbreviation is next to the division symbol, spacing rules for abbreviations next to operation signs are applied. See Lesson 4.

⋙ $6\overline{)1}$ ft. 5 in.



The numeric indicator is not used in a spatially arranged problem.

⋙ $\overline{1}$ ft. 5 in. (6

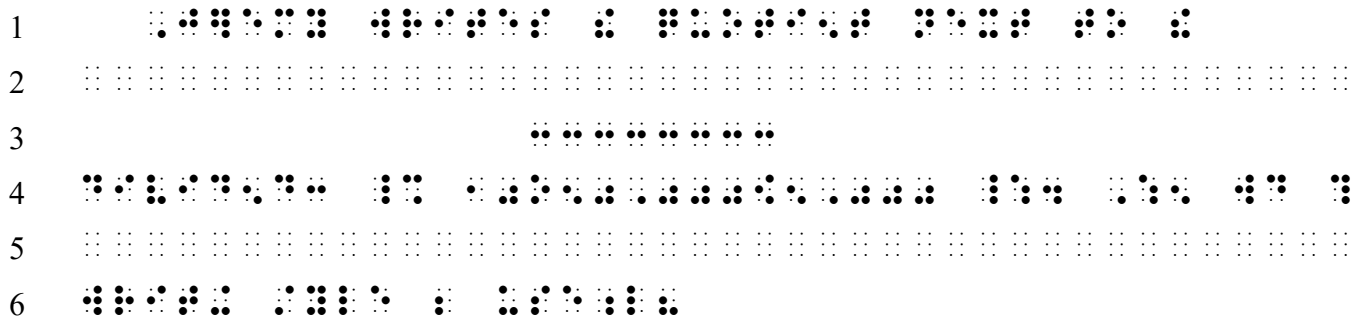


A space is required between an abbreviation and a sign of operation, in this case "in." and the (curved) division sign.

10.13.4 **Embedded Division Problem.** A blank line must precede and follow a spatially arranged problem, even when the problem is embedded within narrative.

Example 10-20

Jeremy writes the quotient next to the dividend: $10\overline{)50,000}(5,000$. When would this writing style be useful?



Lines 2 and 5: Blank lines are required before and after an embedded spatial math expression.
Line 4: Alignment: The divisor and dividend are placed on the same line as the text.
Line 4: Code switch indicators follow the rules for an embedded math expression. In this example, the two switches fit on the same line as the math.

10.13.5 **Review of Format for Spatial Arrangements**

- A blank line is required above and below a spatial arrangement.
- The numeric indicator is not used. (There are some exceptions.)
- Side-by-side arrangement is allowed.
- An identifier associated with a spatial problem is positioned according to rules applying to the type of arrangement, not necessarily at the top line of the problem.

10.13.6 **Long Division.** In a long division problem, multiples are written below the dividend. A series of subtraction problems are performed, giving partial remainders and, finally, a remainder. The long division portion of the problem is aligned for subtraction.

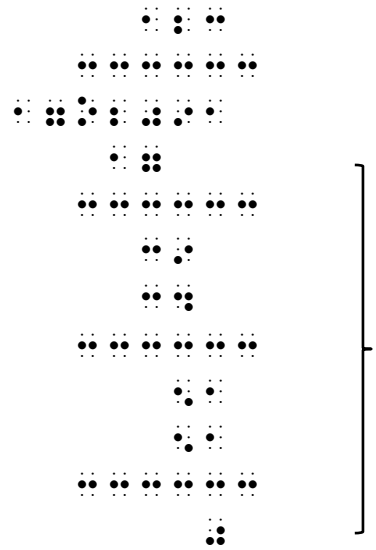
```
   123
17)2091     dividend
  17        multiple (17×1)
  ---       partial remainder
   39       multiple (17×2)
   34       partial remainder
  ---       multiple (17×3)
    51      remainder
    51
    ---
     0
```

subtraction problems

a. **Alignment.** The components of the problem are aligned in the same manner as they are aligned in the print copy. All separation lines must be the same length, regardless of their relative lengths in the print copy.

Example 10-21

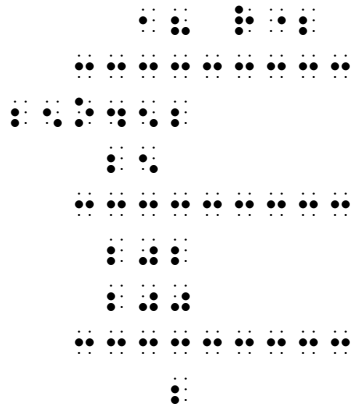
$$\begin{array}{r} 123 \\ 17\overline{)2091} \\ 17 \\ \hline 39 \\ 34 \\ \hline 51 \\ \underline{51} \\ 0 \end{array}$$



aligned for subtraction

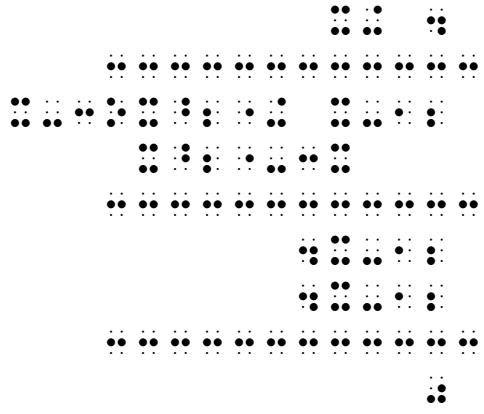
Example 10-22

$$\begin{array}{r} 18\text{r}2 \\ 25\overline{)452} \\ 25 \\ \hline 202 \\ \underline{200} \\ 2 \end{array}$$



Example 10-23

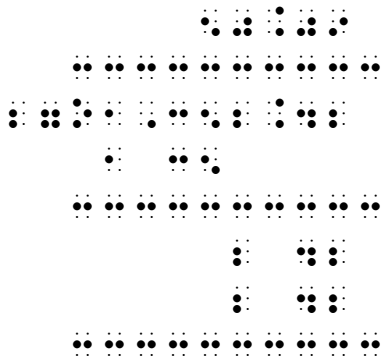
$$\begin{array}{r} x+4 \\ x-3\overline{)x^2+x-12} \\ x^2-3x \\ \hline 4x-12 \\ \underline{4x-12} \\ 0 \end{array}$$



b. **Blank Cells.** When commas or decimal points occur in a dividend, corresponding blank cells are inserted in the body of the division problem in order to achieve alignment. Spaces are not inserted in separation lines.

Example 10-24

```
     50.09
27)1,352.42
   1 35
     242
     242
```



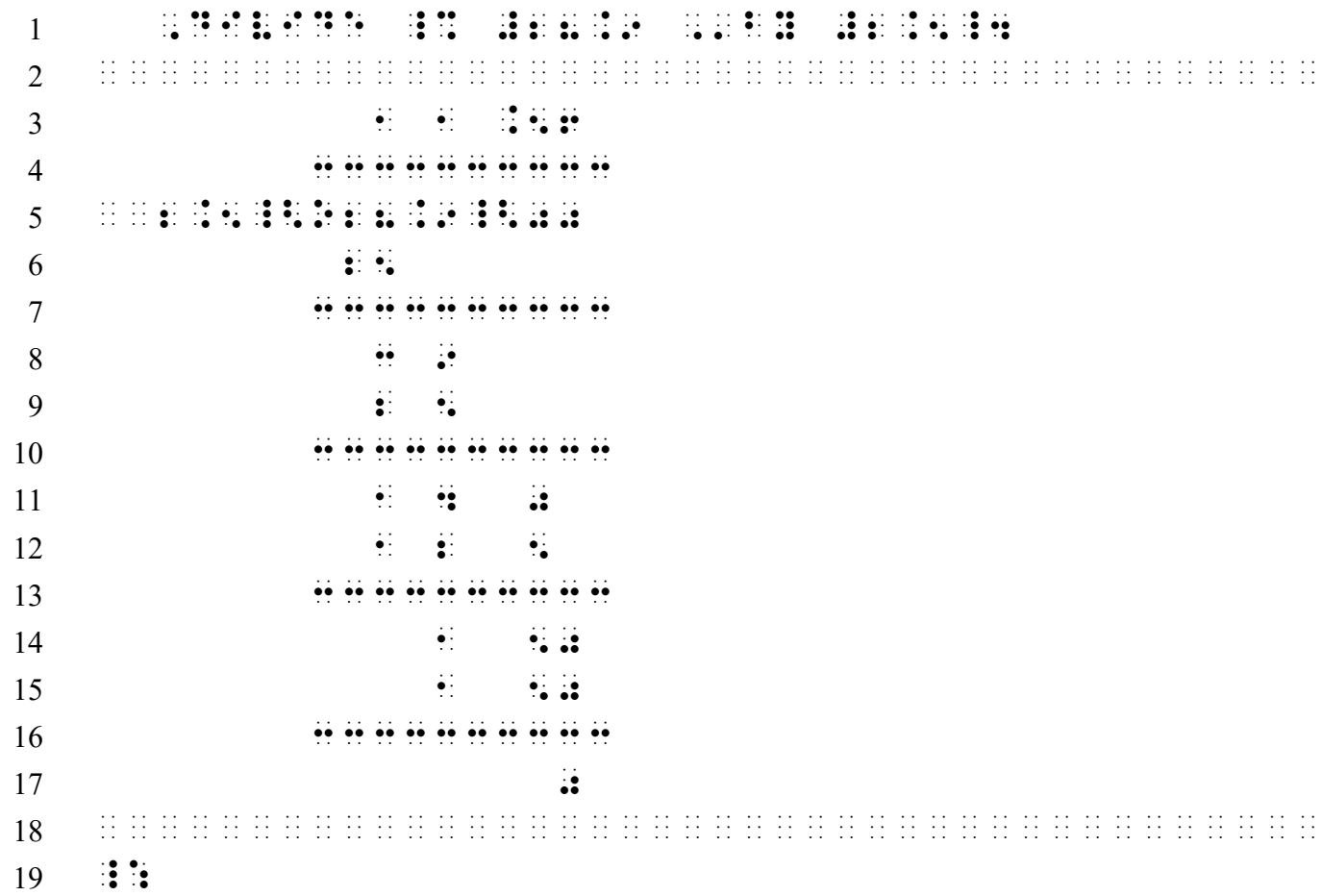
c. **Caret.** If a caret occurs in a dividend, corresponding blank cells are left throughout the body of the division problem, except in the separation lines. Two cells are required for the caret. The decimal point replacing the caret in the quotient is aligned in the right-hand cell.

 Caret ^

Example 10-25

Divide 28.9 by 2.5.

```
      11.56
2.5^)28.9^00
    25
    ---
     39
     25
    ---
     140
     125
    ----
      150
      150
     ---
        0
```

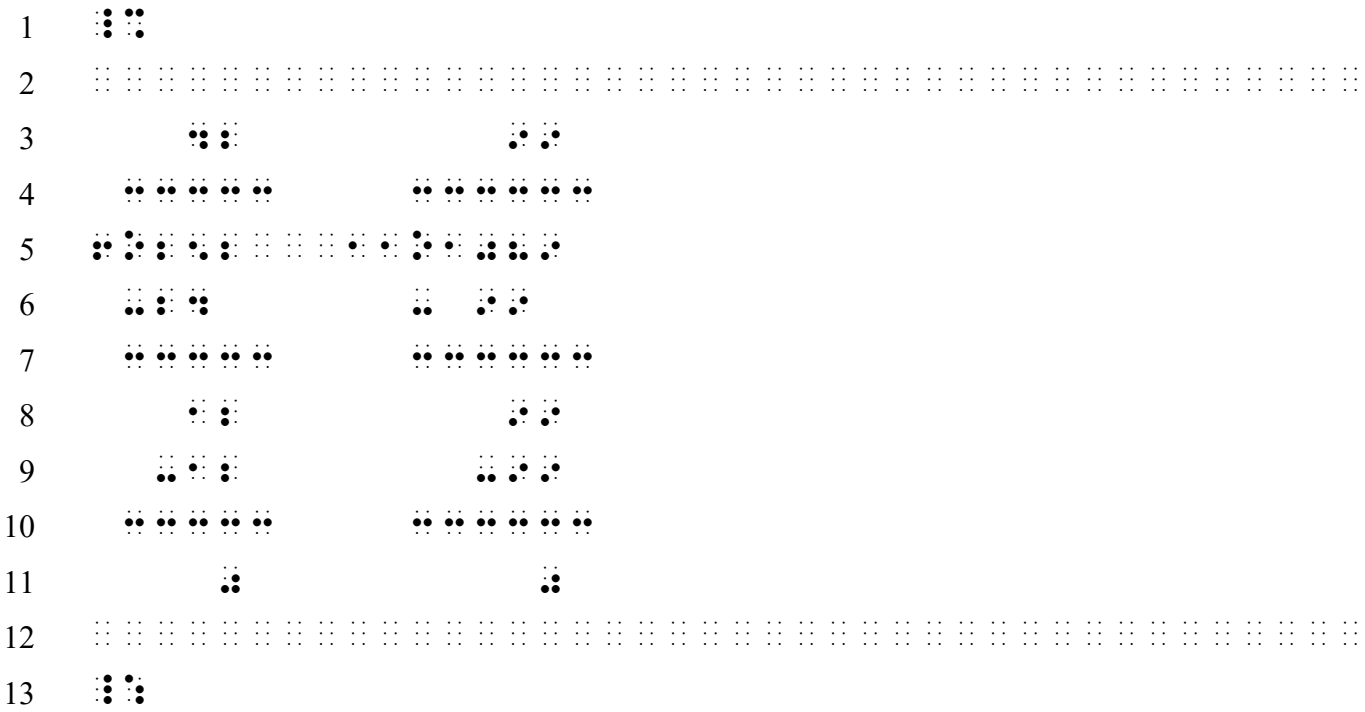


Line 5: The leftmost braille symbol in the displayed spatial arrangement is in cell 3, following margin rules for math displayed to 3-1 text.

d. **Minus Sign.** A minus sign appearing in the long division portion of the problem is placed according to the rules of spatially arranged subtraction problems—one cell to the left of the leftmost numeric symbol that appears above its separation line. If a minus sign appears on the first line below the dividend, the symbol will be placed in the same column as the division symbol. In that case, the following separation line will begin in the same column.

Example 10-26

```
      42          99
   6)252      11)1089
    −24         −99
     12          99
    −12         −99
      0           0
```



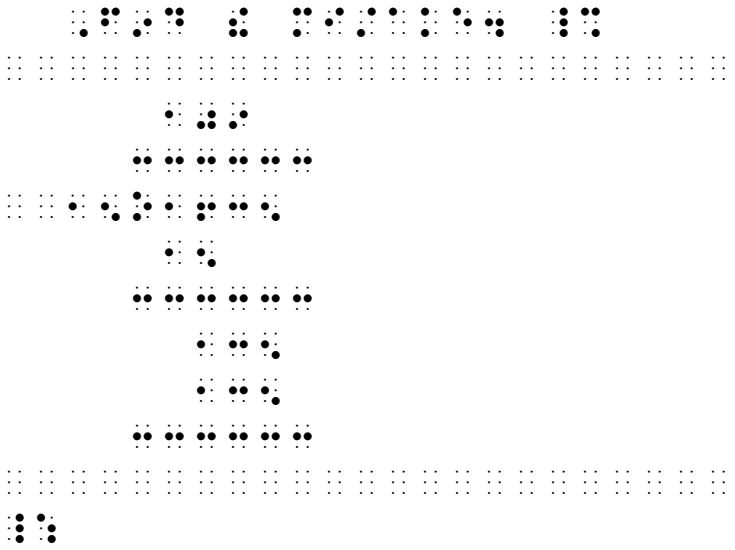
Line 5: According to the rules regarding side-by-side spatial arrangements, three blank cells come between any symbol on any line of the first problem and any symbol on any line of the second problem, not counting separation lines.

e. **Intentional Misalignment.** If the quotient has been intentionally misaligned as an exercise for the student, the same misalignment is shown in the transcription.

Example 10-27

Find the mistake.

```
   109
15)1635
   15
   ---
    135
    135
```



The leftmost character in the spatial arrangement is placed in the appropriate cell for displayed material (cell 3).

PRACTICE 10D

Instructions: Treat the "seven other ways" problems as side-by-side displayed problems. Place each dividend in the same row across the braille line.

Here are seven other ways to write $11\overline{)2233}$ or "2233 divided by 11 equals 203".

$\frac{203}{2233(11}$ $\qquad$ $11/\overline{2233}$ $\qquad$ $11|\overline{2233}$

$\frac{2233\ |\ 11}{203}$ $\qquad$ $\frac{2233(11}{203}$ $\qquad$ $11)\overline{2233}$

$11)\overline{2233(203}$

What is $11\overline{)2234}$? Use long division.

$$\begin{array}{r} 203\ \text{R}1 \\ 11\overline{)2234} \\ \underline{22} \\ 34 \\ \underline{33} \\ 1 \end{array}$$

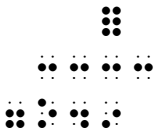
10.14 Omissions in Spatial Division Problems

As noted in Lesson 9, only the general omission symbol is used to show an omission within a spatial arrangement, regardless of the print sign used. The one-cell symbol allows the transcription to maintain alignment.

⠻ General Omission Symbol

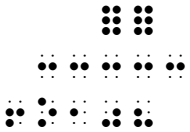
Example 10-28

$$7\overline{)49}\quad\square$$



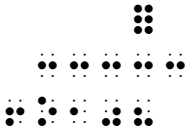
Example 10-29

$$6\overline{)108}\quad??$$



Example 10-30

$$6\overline{)108}$$

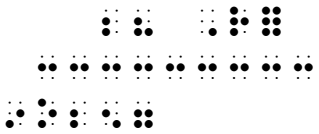


When the print omission sign spans more than one column in the arrangement, the general omission symbol aligns beneath the rightmost character of the omission. (Review Section 9.19.2 in Lesson 9.)

a. When a remainder is to be filled in, a multipurpose indicator is not needed following the letter "R" because the general omission symbol is nonnumeric, even though it may represent a numeral.

Example 10-31

$$9\overline{)257}\quad 28\ R\square$$

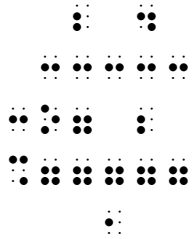


10.15 Regrouping in Division

The regrouping indicator is transcribed above or below the dividend, depending on the position of the separation line. The first cell of each regrouping indicator tells whether to read the numbers above or below the line. The regrouping indicator is one cell longer on the left than the separation line. A blank space is left in the dividend and in the quotient where necessary to accommodate a regrouping number. No blank spaces are left in the regrouping line.

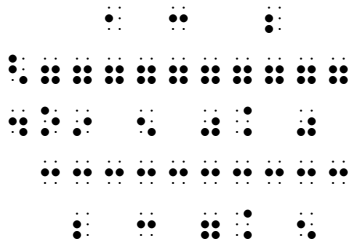
a. When the separation line is printed above the dividend, the regrouping indicator for numbers *below* the arrangement is transcribed below the dividend.

⩾ $\begin{array}{r} 24 \\ 3\overline{)7^{1}2} \end{array}$



b. When the separation line is printed below the dividend, the regrouping indicator for numbers *above* the arrangement is transcribed above the dividend.

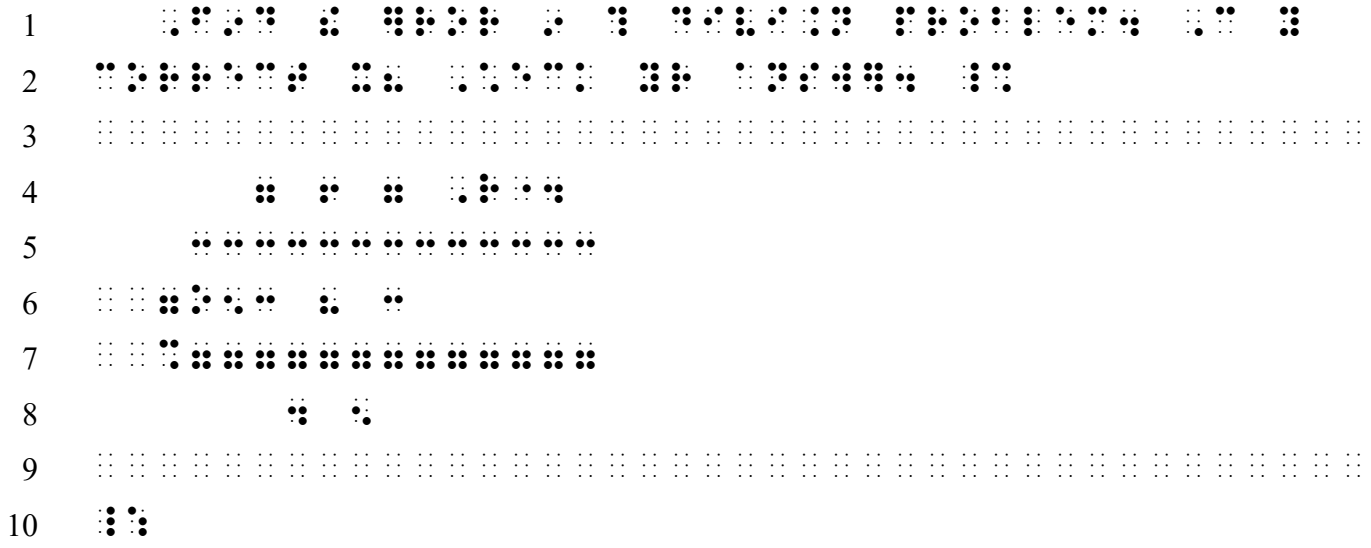
⩾ $\begin{array}{r} 4)9^{1}5^{3}0.^{2}0 \\ \hline 237.5 \end{array}$



Example 10-32

Find the error in this division problem. Can you correct it? Check your answer.

$$\begin{array}{r} 767\ \text{R}4 \\ \hline 7\overline{)53_{4}8_{5}3} \end{array}$$



10.16 Cancellation in Long Division

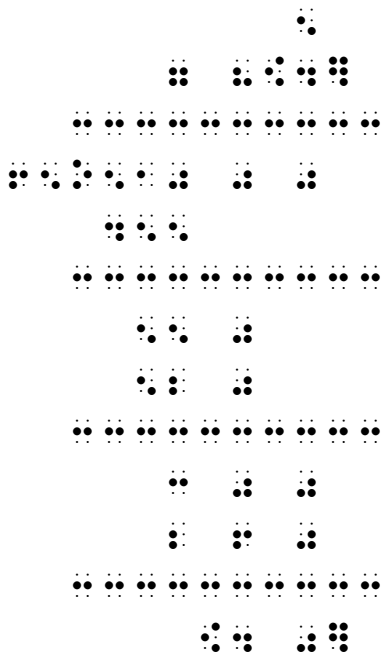
Cancellation was introduced in Lesson 9. Canceled material is enclosed in braille cancellation indicators.

⠰⠣	Opening Cancellation Indicator
⠰⠜	Closing Cancellation Indicator

The canceled material must exactly represent what is canceled in print. To ensure a clear column above and below the cancellation indicators, blank cells are inserted. Alignment of digits must match the print layout.

Example 10-33

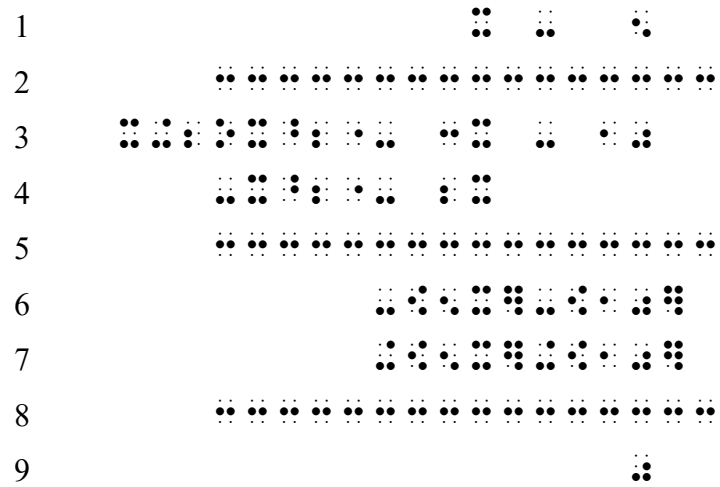
```
      5
    78~~4~~
65)51000
   455
   ---
    550
    520
    ---
     300
     260
     ---
     ~~40~~
```



Notice the column of clear cells below the opening cancellation indicator on the second line and above the opening cancellation indicator on the last line.

Example 10-34

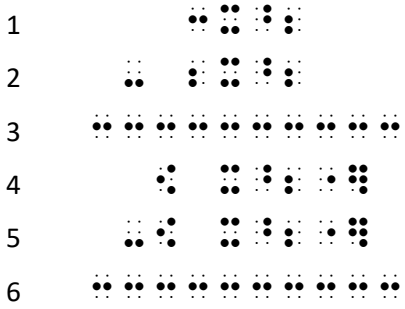
$$\begin{array}{r} x - 5 \\ x+2 \overline{)\, x^2 - 3x - 10} \\ \underline{-x^2 - 2x} \\ \cancel{-5x} - \cancel{10} \\ \underline{+\cancel{5x} + \cancel{10}} \\ 0 \end{array}$$



Notice the alignment of components: x's with x's; indicators with indicators. Digits are aligned by place value. Spaces are inserted throughout the problem to ensure that no numeral or operation sign is in the same column as a cancellation indicator.

Special Note: When aligning parts of a polynomial problem with cancellation, do not separate a coefficient from its variable, as illustrated in this portion extracted from a long division problem.

$$\gg \quad \begin{array}{r} 3x^2 \\ \underline{-2x^2} \\ \cancel{x^2} \\ \underline{-\cancel{x^2}} \end{array}$$



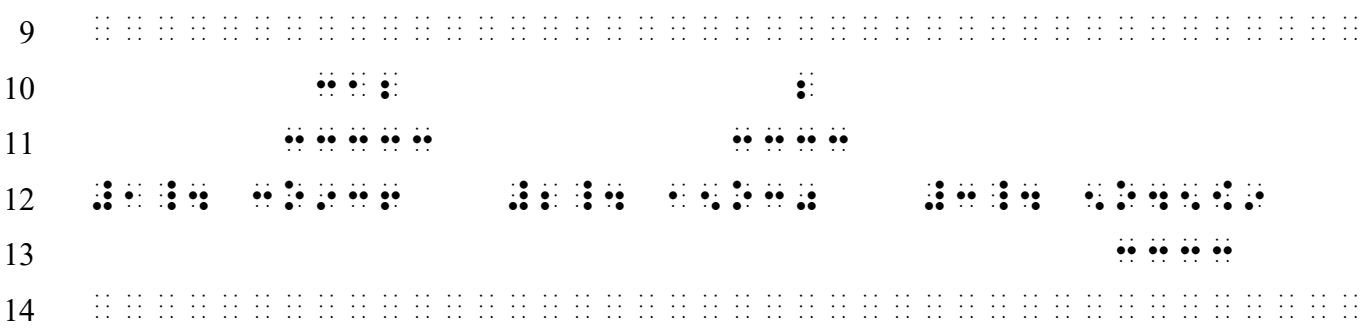
Lines 4-5: A space is inserted where the coefficients are located above the separation line.

10.17 Placement of Identifiers with Spatial Division

An identifier, if present, is placed on the line with the dividend. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines. When arranged side by side across the width of the page, no symbol of one spatial arrangement or its identifier may be less than three cells distant from any symbol on any line of a neighboring arrangement or its identifier, except at the ends of separation lines.

Example 10-35

1. $3\overline{)936}$ $\overline{312}$ 2. $15\overline{)30}$ $\overline{2}$ 3. $\underline{5)45}(9$



10.18 Other Layouts

Other types of division problems will be taught in a later lesson.

PRACTICE 10E

Instructions: Do not use side-by-side layout in this transcription.

Fill in the boxes with the correct numeral.

(a) $6\overline{)23^{5}5^{1}813}$ $\overline{3\,9\,302\,R\square}$

(b) $25\overline{)80}$ $\overline{3\frac{1}{5}}$ $\overline{5}$ $\overline{3}$ $\overline{25}$ $\underline{75}$ $\square$

(c) $4\overline{)19}$ $\overline{\square\,R3}$

10.19 Summary of the Use and Nonuse of the Numeric Indicator

It may be helpful to summarize the use and nonuse of the Nemeth numeric indicator studied so far. This is not an all-inclusive list.

The word "numeral" includes a number that begins with a decimal point or a number that begins with a minus sign (a negative number) as well as a simple number consisting of one or more digits. These rules also apply to nondecimal digits such as T and E.

10.19.1 **Use of the Numeric Indicator. [NC Rule 3.3]** A numeric indicator is needed in the following circumstances.

a. when a Nemeth numeral begins a braille line or is preceded by a space.

b. at the beginning of the runover line of a long (divided) numeral.

c. when a Nemeth numeral immediately follows a punctuation mark. (A grouping sign is not a mark of punctuation. See subitem d. regarding the hyphen, which requires special attention.)

d. after a hyphen which connects a Nemeth numeral to a word, an abbreviation, or a punctuation mark. (A grouping sign is not a punctuation mark.)

e. when a Nemeth numeral follows a mathematical asterisk, crosshatch, paragraph mark, or section mark.

f. when a Nemeth numeral is in nonregular type.

g. when there is change back to regular type from nonregular type within the same Nemeth numeral.

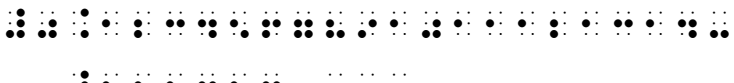
The following samples illustrate points a-g.

(a) .5 −1 t4e

0.333 ... 3 ...

−.7

(b) 0.12345678910 (etc.)



(c) "3.4"

2:30 + 1:15 =

(d) 1.5 million-2.5 billion

4K-5K

6.3?-6.8

(e) $3 * 9 = 27$ 

(f) **183** 

(g) 14**6**2**5** 

10.19.2 **Nonuse of the Numeric Indicator. [NC Rule 3.4]** A numeric indicator is not needed in the following circumstances, as long as the numeral is in regular type.

h. when a numeral is unspaced from and follows a mathematical character or a slash.

i. when a numeral is unspaced from and follows a Nemeth indicator.

j. when a numeral is unspaced from and follows a left grouping sign.

k. when a numeral follows a hyphen which follows a numeral or other mathematical symbol.

l. when a numeral in regular type is part of an "enclosed list".

m. when a numeral is partitioned into segments with spaces.

n. when a numeral is part of a spatially arranged addition, subtraction, multiplication, or division problem.

The following samples illustrate points h-n.

(h) $-7 + 12 - 3 = +2$ 

$1.5K-$2.5B 

$2/3$ 

(i) x^2 $^{-}2$ R5 

$\frac{3}{4}$ $\sqrt[3]{8}$ 

(j) $(-4$ and $-5)$

$(0 = x)$

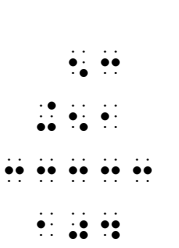
(7%)

(k) 3.4-3.8 cc

80%-90%

(l) {.5, .7, .9, 1.1}

(m) 987 656 000

(n)
$$\begin{array}{r} 53 \\ +51 \\ \hline 104 \end{array}$$


10.20 Review of Rules for Signs of Grouping

A review of the cited sections is recommended.

a. Inside the switches, grouping symbols (parentheses, brackets, braces) are not considered to be punctuation; they are signs of grouping. [2.9]

b. The numeric indicator is not used before a numeral which immediately follows a grouping symbol. [2.10]

c. Nemeth grouping symbols are punctuated mathematically. [2.12.1]

d. Inside the switches, grouping symbols are transcribed using the Nemeth symbols even when they have no mathematical meaning. [2.12.b]

e. When an isolated math symbol is enclosed between parentheses, brackets, or braces, the paired grouping symbols are transcribed in Nemeth. [2.12]

f. Paired grouping symbols must be transcribed in the same code. [2.13]

g. The English-letter indicator is not used when a single English letter or a Roman numeral is entirely enclosed between signs of grouping. [3.12.2, 4.6] This rule applies to mathematical "single letters" only, not to single-letter abbreviations. [4.21]

h. The appropriate alphabetic indicator must be used with any letter from the German, Greek, Hebrew, or Russian alphabets. [4.10.1] This rule applies even when non-English letters are enclosed between, or in contact with, signs of grouping.

i. A sequence of mathematical items enclosed in signs of grouping (an "enclosed list") must use the Nemeth grouping symbols. [4.17]

j. When a single English letter or a Roman numeral is in direct contact with only one sign of grouping, the English-letter indicator is or is not used as though the grouping sign was not present. If the grouping sign carries a prime or other modifying symbol, the English-letter indicator is not used. [4.18]

k. When a grouping symbol appears on the baseline level and a level indicator is currently in effect, the baseline indicator is placed before the sign of grouping. If no subscript indicator is used, a return to the baseline is implied. [6.21]

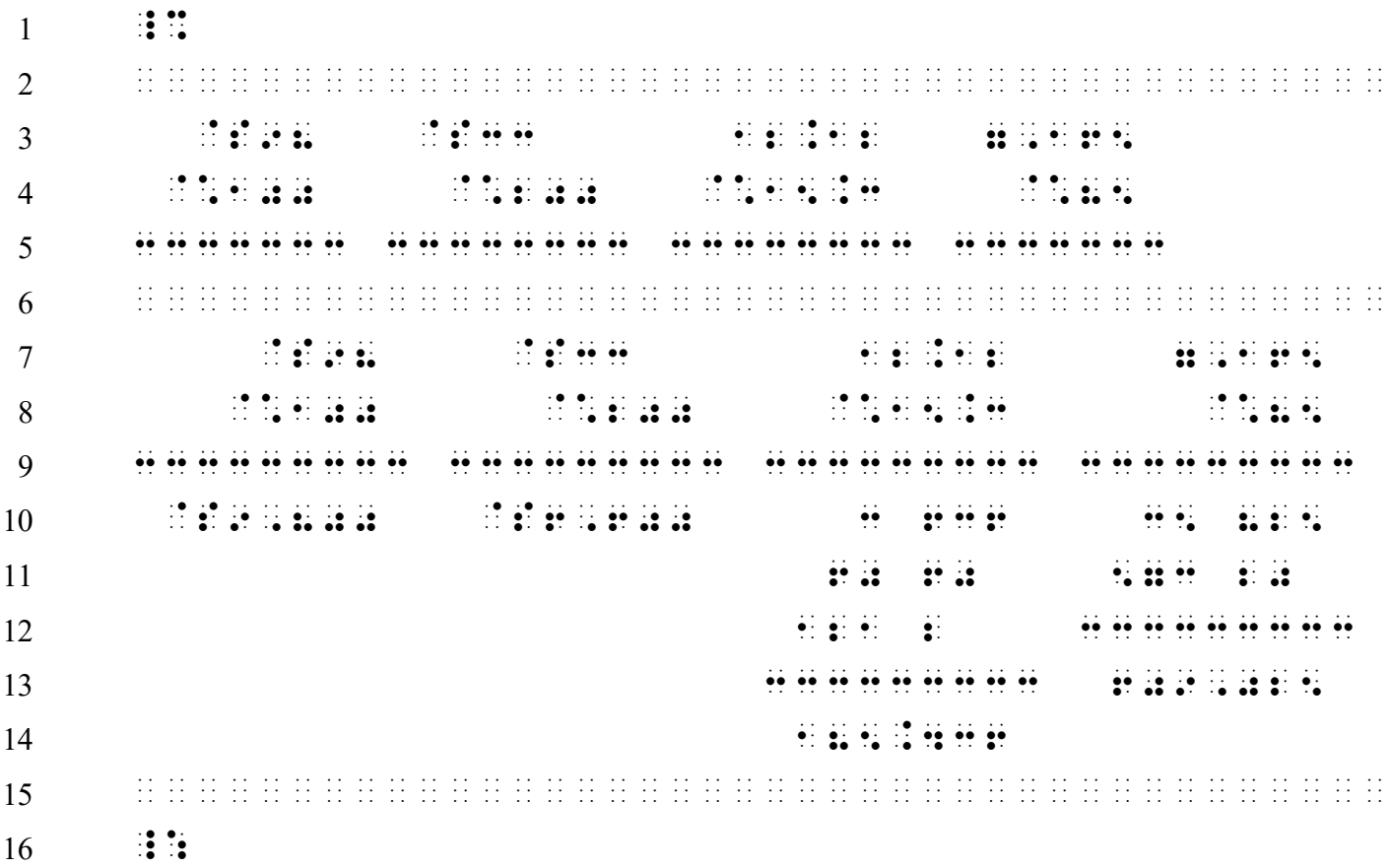
l. When brackets or vertical bars are printed in mathematically significant boldface, dots 456 are used before the grouping symbol. [7.9.3]

For further practice, see Addendum 1—Reading Practice.

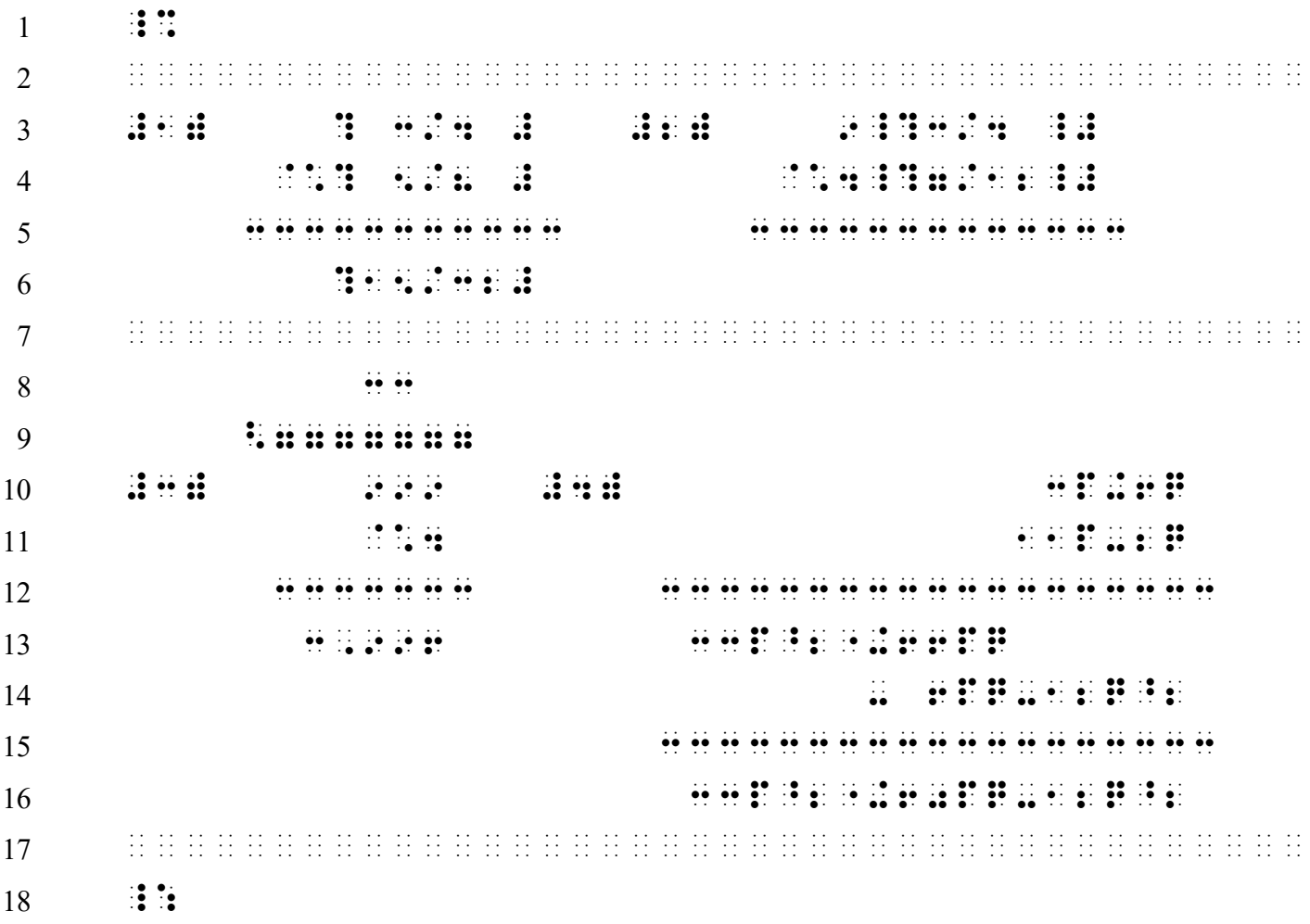
Submit Exercise 10 to your instructor.

ANSWERS TO PRACTICE MATERIAL

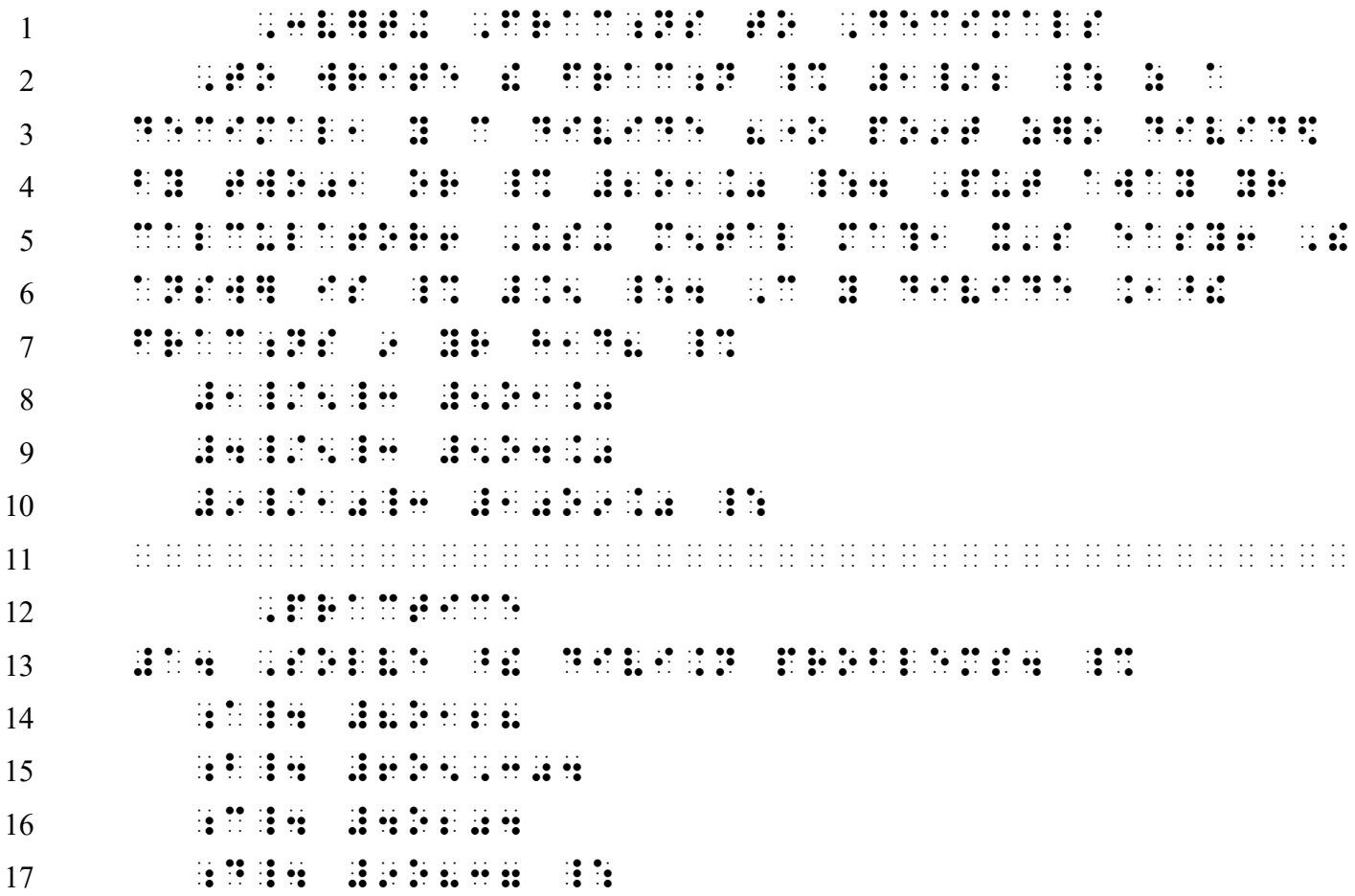
PRACTICE 10A



PRACTICE 10B

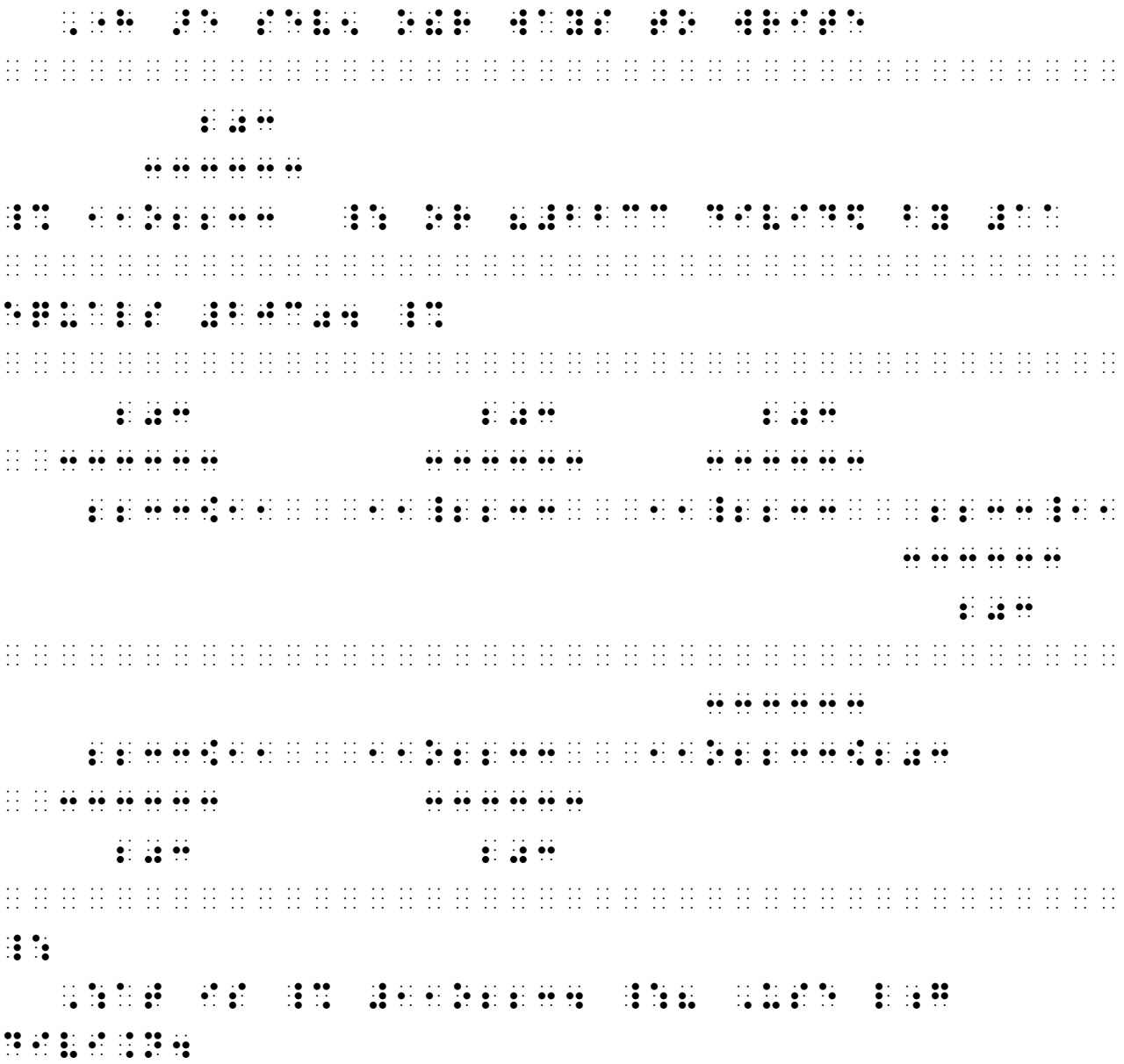


PRACTICE 10C



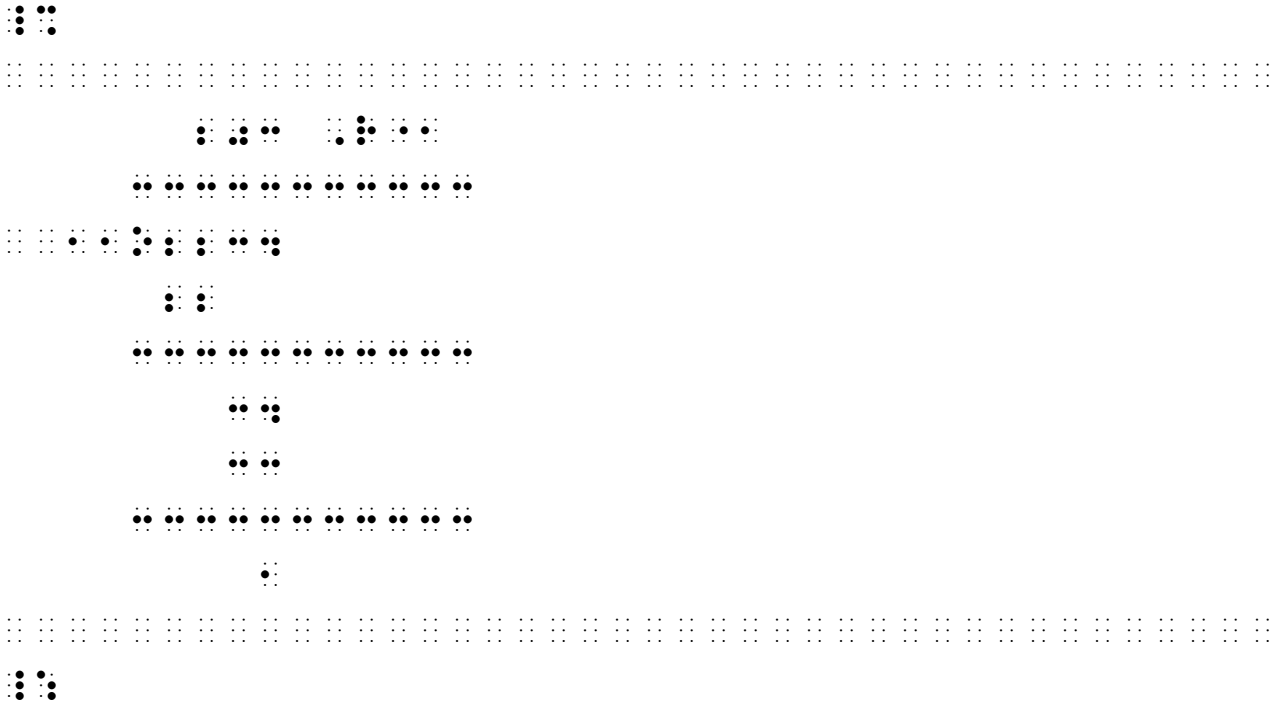
Lines 14-17: Each subdivision begins on a new line in cell 3 even though the problems are arranged side by side in print.

PRACTICE 10D



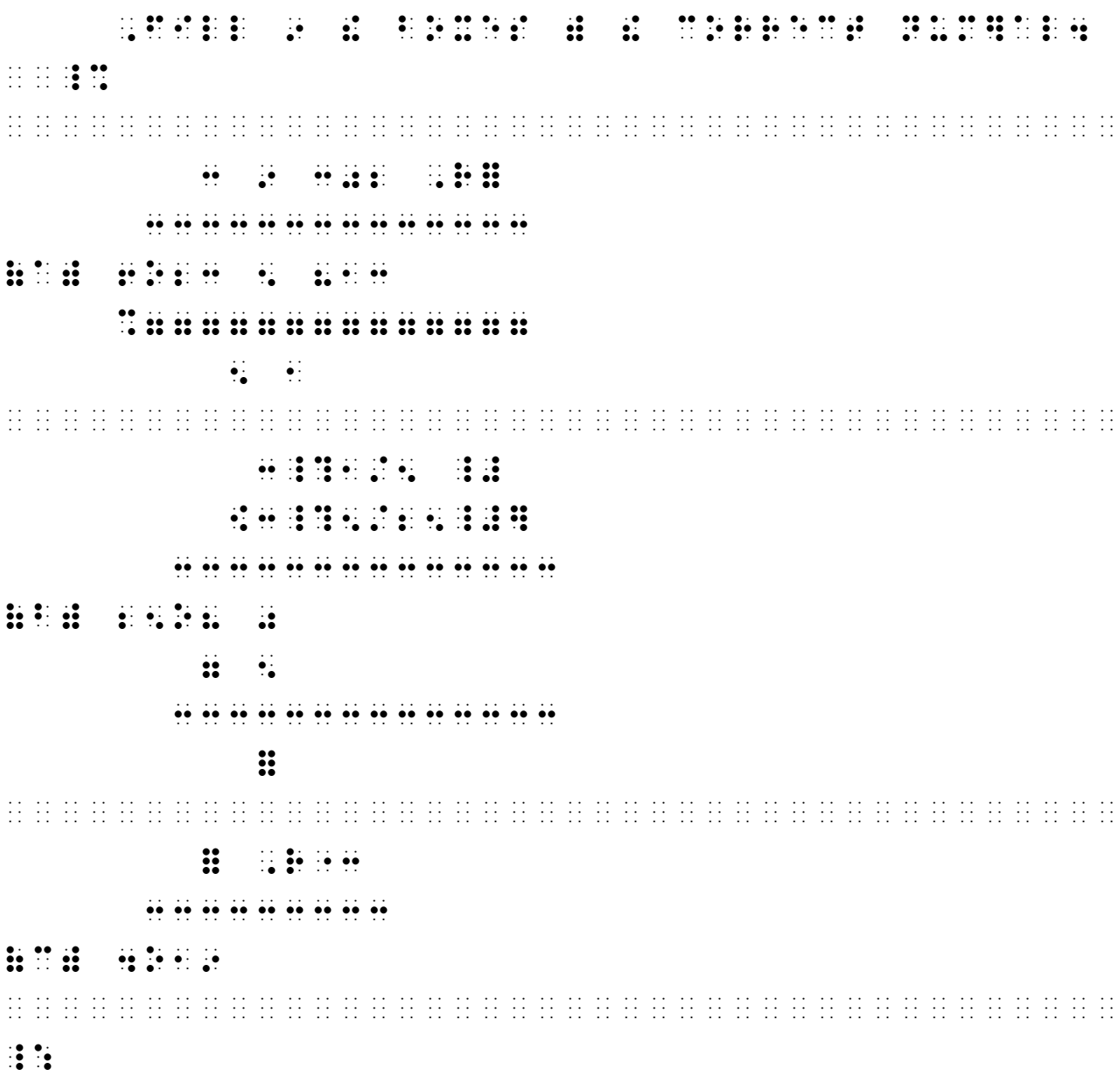
Lines 2 and 6 are blank because a spatial arrangement must be preceded and followed by a blank line, even when embedded within a narrative paragraph.
Line 5: The code switch indicators are placed on the main line of the embedded math problem.
Lines 10 and 17: The leftmost symbol of the first arrangement in each row is placed in cell 3 because it is displayed to a 3-1 paragraph.
Lines 11 and 16: Three blank cells between symbols in neighboring arrangements is the minimum number allowed (separation lines excluded).
Line 20: The Nemeth Code terminator is placed in cell 1 following the blank line after the completion of the spatially arranged material.
Line 21: The next paragraph begins on a new line.

PRACTICE 10D, continued



Line 1: The opening switch is placed on the same braille page as the math to which it applies. It is placed in cell 1 because a spatial arrangement follows.
Line 5: The leftmost symbol is placed in cell 3 because it is displayed to a 3-1 paragraph.
Line 13: The Nemeth Code terminator is placed in cell 1 following the blank line after the completion of the spatially arranged material.

PRACTICE 10E



Line 1: The instructions begin in cell 5.
Line 2: There is not room for the opening Nemeth Code indicator on line 1. It is placed in the runover cell for instructions (cell 3) because an itemized spatial arrangement follows. See Section 9.30 in Lesson 9.
Line 4: See Section 10.14.a regarding the transcription of the remainder notation in problem (a).