LESSON 15

Format

LESSON PREVIEW

In this lesson we discuss what to do when a mathematical expression will not fit on one braille line. Rule 26.2 of the Nemeth code gives basic instructions for selecting runover sites, but application of the rule can be daunting given the wide variety of layouts encountered in a transcription. Lesson 15 explores many typical scenarios. The examples serve as a good review of other aspects of the Nemeth code.

MATHEMATICAL EXPRESSIONS REQUIRING RUNOVERS

[NC 26.2]

15.1 Review

Above all else, a mathematical expression must not be divided between lines if it fits one braille line within the current margins. Before considering division options, check whether the math will fit on one line …

… if you place related code switch indicators on a different line, will the math fit on one line?

… if you start the math on a new line following its related identifier, will it fit on one line?

… if you move the math to a line that does not contain a page number, will it fit on one line?

Here is a review of items that must not be divided and a summary of runover rules already covered in the lessons.

15.1.1 **Symbols to Keep Together.** The components of the following symbols must not be divided between braille lines.

- A symbol of operation using plus and minus (Lesson 5)
- A symbol of comparison compounded vertically or horizontally (Lesson 5)
- A shape symbol with structural or interior modification (Lesson 11)
- A keystroke construction (Lesson 11)
- Superposed symbols (Lesson 13)
- Tally marks belonging to the same group (Lesson 13)

15.1.2 **Expressions to Keep Together.** The following expressions must not be divided between braille lines, even if divided in print.

- A hyphenated expression of which one component requires Nemeth. (Lesson 2)
- An abbreviation and its related numeral or letter. (Lessons 3 and 4)
- A shape symbol and its name (numeral, letter, or sequence of letters). (Lesson 11)
- The components of an expression modified according to the five-step rule. (Lesson 12)
- A function name (or its abbreviated form) and its argument. (Lesson 14)
- A two-part function name. (Lesson 14)

15.1.3 When a long expression does not fit on the braille line, use as much of the first line as possible before dividing at an appropriate location. A hyphen is not inserted, with one exception (a long numeral). The new line begins in the runover cell of the current format. Here is a list of runover rules already studied:

- **Long Numeral.** If a long numeral will not fit on one line, divide after a comma if a comma is present. A hyphen is inserted at the point of division. If the numeral does not contain a comma, the hyphen may be inserted after any digit. The numeric indicator is restated before the first digit of the continuation of the numeral on the next line. (Lesson 1)
- **Enclosed List.** If an enclosed list will not fit on one line, divide after a comma used to separate the items. (Lesson 4)
- **Linked Expression.** If a linked expression will not fit on one line, divide before a link. It is not necessary to divide at every link unless the layout follows that of a nested linked expression. (Lesson 8)
- **A Series of Keystrokes.** If a series of keystrokes will not fit on one line, division is made after any item in the keystroke string but not within a keystroke. (Lesson 11)
- **Individual Mathematical Expressions.** When two or more math expressions occur between the same code switch indicators, the line may wrap at the space between the expressions, even if the entire Nemeth portion could fit on one line. (Lesson 1)

A SYSTEMATIC APPROACH

15.2 Mathematical Units

The braille transcriber has only 40 cells available on a line at most – as few as 30 cells in some layouts. We often encounter a mathematical expression that will not fit on the current line. The margins in place at the time should not be changed in order to accommodate a long expression. Instead, the math will need to be divided.

A long or complicated mathematical expression can be organized into a series of mathematical units by following the procedures presented in this lesson. When the transcriber applies these principles, the reader is able to mentally reassemble the expression. On the other hand, a poorly divided expression will hinder the reader's understanding of the mathematics. Shrewd application of these guidelines can be properly rendered even if the transcriber is unfamiliar with the particular mathematics.

We will now examine how to choose runover sites in order to keep the following mathematical units intact:

- the anchor in a linked expression
- a link in a linked expression
- a fraction or, if unavoidable, its numerator or its denominator
- a grouped expression
- a base and its exponent; a subscript and its related item
- a radical expression
- a modified expression or, if unavoidable, a modifier within the modified expression

When choosing a division site, start with step a.

Step a Divide before a comparison sign on the baseline. See Section 15.3. (The comparison sign cannot be within a mathematical unit.)

If there is no such comparison sign or if there is still not room, apply step b.

Step b Divide before an operation sign on the baseline. See Section 15.4. (The operation sign cannot be within a mathematical unit.)

If there is no such operation sign or if there is still not room, choose from step c or step d.

Step c Divide before a mathematical unit. See Section 15.5.

Step d Divide after a termination indicator. See Section 15.6.

Each step in the systematic approach will now be examined. Take your time analyzing each example. Isolated examples without narrative do not include code switch indicators in order to focus on layout.

15.3 Step a: Divide Before a Comparison Sign on the Baseline

As you learned in Lesson 8, a linked expression is comprised of an anchor followed by one or more links and division is made at a link (before the comparison symbol) when the entire expression will not fit on one braille line.

Keep these points in mind as you study the examples in this section.

- Determine whether the expression can fit on one line using tactics given in Section 15.1.
- If the math will not fit, use as much of the line as possible before dividing.
- The comparison sign at which the division is made must be on the baseline of writing.
- The link begins in the appropriate cell according to the current format. (Example 15-1) *Note:* The print copy may divide after a comparison sign, but the braille transcription divides *before* the comparison symbol.
- Even if the anchor consists of only one letter or number, if the link will not fit on the line with the anchor, the line is divided after the anchor. (Example 15-2)
- If the expression contains more than one link, it is not necessary to divide at every link. (Example 15-3)

  Exceptions: A nested linked expression is divided at every link (see Example 8-37 in Lesson 8); if the anchor or any link is divided, divide at every link (to be covered in Section 15.4.2).
- When a line begins with a comparison symbol, the transition to a new braille line terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines. (Example 15-4-b)
- In itemized formats, if an anchor will not fit on the line with its identifier but it fits on the next line starting in the runover cell, put it there in order to keep the anchor intact. The identifier will then be the only item on the first line. (Examples 15-4-a and 15-5)

- Unless unavoidable, a sign of comparison within a mathematical unit is not a suitable division site. (Examples 15-6 and 15-7)

A few examples are shown here as a review. Commentary will walk you through the process. Embedded material is assumed to be within a 3-1 paragraph. Displayed material is assumed to be displayed to a 3-1 paragraph, beginning in cell 3 with runovers in cell 5.

Example 15-1

THE LINKED EXPRESSION REQUIRES 43 CELLS

$1{,}778 + 1{,}294 + 865 + 905 + 2574 + 485 + 100 \;>\; 8{,}000$

(15-1-a, embedded)

⠼⠂⠠⠶⠶⠦⠬⠂⠠⠆⠔⠲⠬⠦⠖⠢⠬⠔⠴⠢⠬⠆⠢⠶⠲⠬⠲⠦⠢⠬⠂⠴⠴
⠨⠂⠀⠼⠦⠠⠴⠴⠴

Starting in cell 1, the anchor (33 cells) fits on one line. The link is placed on the next line in the runover cell of the current 3-1 format (cell 1).

(15-1-b, displayed)

⠀⠀⠼⠂⠠⠶⠶⠦⠬⠂⠠⠆⠔⠲⠬⠦⠖⠢⠬⠔⠴⠢⠬⠆⠢⠶⠲⠬⠲⠦⠢⠬⠂⠴⠴
⠀⠀⠀⠀⠨⠂⠀⠼⠦⠠⠴⠴⠴

Starting in cell 3, the anchor (33 cells) fits on one line. The link is placed on the next line in the runover cell of the current 3-5 format (cell 5).

Example 15-2

THE LINKED EXPRESSION REQUIRES 42 CELLS

(embedded)

The degree of the polynomial $y = 6x^3 - 2x^2y^1 + x^2y^2 - 3x^2y^2 + z$ is 4.

⠀⠀⠠⠮⠀⠙⠑⠛⠗⠑⠑⠀⠷⠀⠮⠀⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇⠀⠸⠩⠀⠽⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠨⠅⠀⠼⠖⠭⠘⠒⠐⠤⠆⠭⠘⠆⠐⠽⠘⠂⠐⠬⠭⠘⠆⠐⠽⠘⠆⠐⠤⠒⠭⠘⠆⠐⠽⠘⠆⠐⠬⠵
⠸⠱⠀⠊⠎⠀⠼⠙⠲

This anchor consists of the letter y and sits alone at the end of line 1 because it does not fit on the same line as its long link.

Example 15-3

EACH LINKED EXPRESSION REQUIRES 41 CELLS

(displayed)

To factor the expression $-2ab + a^2 + b^2$,

Jared wrote: $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$
Dom wrote: $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a - b)^2$

Both solutions are correct. Explain.

```
1 ⠀⠀⠠⠞⠕ ⠋⠁⠉⠞⠕⠗ ⠮ ⠑⠭⠏⠗⠑⠎⠨⠝
2 ⠸⠩ ⠤⠼⠆⠁⠃⠬⠁⠘⠆⠐⠬⠃⠘⠆ ⠸⠱⠂
3 ⠀⠀⠠⠚⠜⠫ ⠺⠗⠕⠞⠑⠒ ⠸⠩ ⠤⠼⠆⠁⠃⠬⠁⠘⠆⠐⠬⠃⠘⠆
4     ⠨⠅ ⠃⠘⠆⠐⠤⠆⠁⠃⠬⠁⠘⠆ ⠨⠅ ⠷⠃⠤⠁⠾⠘⠆ ⠸⠱
5 ⠀⠀⠠⠙⠕⠍ ⠺⠗⠕⠞⠑⠒ ⠸⠩ ⠤⠼⠆⠁⠃⠬⠁⠘⠆⠐⠬⠃⠘⠆
6     ⠨⠅ ⠁⠘⠆⠐⠤⠆⠁⠃⠬⠃⠘⠆ ⠨⠅ ⠷⠁⠤⠃⠾⠘⠆ ⠸⠱
7 ⠠⠃⠕⠹ ⠎⠕⠇⠥⠰⠝⠎ ⠜⠑ ⠉⠕⠗⠗⠑⠉⠞⠲ ⠠⠑⠭⠏⠇⠁⠔⠲
```

Lines 1-2: The paragraph begins in cell 3 with runovers in cell 1.
Line 2: The embedded math and its two code switches are placed on the same line.
Line 3: The displayed expression begins in cell 3. Since the entire expression—the anchor and its two links—will not fit on one line, the anchor is placed on this line. Division is made before the comparison sign of the first link.
Line 4: The displayed expression continues in the runover cell (cell 5). Both links fit on this line. It is not necessary to divide this linked expression at every comparison sign.
Lines 5-6: Same as lines 3-4.
Line 7: The paragraph continues in the runover cell (cell 1).

Example 15-4

THE LINKED EXPRESSION REQUIRES 37 CELLS

(15-4-a, itemized)

13. $16x^4 + 81y^4 = (4x^2)^2 + (9y^2)^2$

```
1 ⠼⠂⠒⠸⠲
2   ⠼⠂⠖⠭⠘⠲⠐⠬⠦⠂⠽⠘⠲ ⠨⠅ ⠷⠲⠭⠘⠆⠐⠾⠘⠆⠐⠬⠷⠔⠽⠘⠆⠐⠾⠘⠆
```

Line 1: The linked expression will not fit on line 1 along with its identifier.
Line 2: The entire linked expression fits on one line, starting in the runover cell of the current format (cell 3). Division is not needed.

(15-4-b, itemized subdivision)

13.
a. $16x^4 + 81y^4 = (4x^2)^2 + (9y^2)^2$

```
1 ⠼⠂⠒⠸⠲
2   ⠰⠁⠸⠲ ⠼⠂⠖⠭⠘⠲⠐⠬⠦⠂⠽⠘⠲⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
3     ⠨⠅ ⠷⠲⠭⠘⠆⠐⠾⠘⠆⠐⠬⠷⠔⠽⠘⠆⠐⠾⠘⠆⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
```

Line 2: Once it is determined that the entire linked expression will not fit on line 3 where only 36 cells are available, the anchor is placed on the line with its associated identifier.
Line 3: The link begins in the current runover cell (cell 5).

Example 15-5

THE LINKED EXPRESSION REQUIRES 34 CELLS

(displayed)

Dividing by $a^2(a^2 - c^2)$,

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1. \qquad (3.3)$$

```
1 ⠀⠀⠠⠙⠊⠧⠊⠙⠬ ⠃⠽ ⠸⠩ ⠁⠘⠆⠐⠷⠁⠘⠆⠐⠤⠉⠘⠆⠐⠾⠠
2 ⠀⠀⠷⠒⠨⠒⠾⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
3     ⠹⠭⠘⠆⠐⠌⠁⠘⠆⠐⠼⠬⠹⠽⠘⠆⠐⠌⠁⠘⠆⠐⠤⠉⠘⠆⠐⠼ ⠨⠅ ⠼⠂⠀⠀
4     ⠸⠱⠲
```

Line 1: The paragraph begins in cell 3. There is no runover.
Line 2: Although the anchor (28 cells) will fit on this line, since the entire expression will fit on line 3, it is not placed here. (Review Section 7.2 in Lesson 7 regarding displayed material with identifiers printed to the right.)
Line 3: The entire linked expression fits on one line, starting in the runover cell of the current format (cell 5).
Line 4: The Nemeth Code terminator and punctuation do not fit on the line above. They are placed in the runover position (cell 5).

15.3.1 A Sign of Comparison Within a Mathematical Unit

Unless unavoidable, a sign of comparison within a mathematical unit is not a suitable division site. (See the list of mathematical units in Section 15.2.)

Example 15-6

THE LINKED EXPRESSION REQUIRES 62 CELLS

(embedded)

$\{x \mid x \text{ is an even integer}\} \nsubseteq \{x \mid x \text{ is an odd integer}\}$

⠨⠷⠭ ⠳ ⠭ ⠊⠎ ⠁⠝ ⠑⠧⠑⠝ ⠊⠝⠞⠑⠛⠑⠗⠨⠾⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠌⠸⠐⠅⠱ ⠨⠷⠭ ⠳ ⠭ ⠊⠎ ⠁⠝ ⠕⠙⠙ ⠊⠝⠞⠑⠛⠑⠗⠨⠾

A grouped expression is a mathematical unit. Within each grouping the vertical bar meaning "such that" is a comparison sign but, since the grouping will fit on one line, it is not divided. The preferred division site in this linked expression is the comparison sign between the groupings (the negated bar under inclusion meaning "is not a subset of").

Example 15-7

THE LINKED EXPRESSION REQUIRES 52 CELLS

(embedded)

$$\sum_{i=0}^{n}\sum_{j=0}^{m}=\lim_{n\to\infty}\underline{\qquad}$$

⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠴⠣⠝⠻⠐⠨⠠⠎⠩⠚ ⠨⠅ ⠼⠴⠣⠍⠻⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠨⠅ ⠐⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠤⠤⠤⠤

A modifier is a mathematical unit. The comparison sign within each of these modifiers is not a suitable division site (equals sign, arrow). The preferred division site in this linked expression is the comparison sign printed on the baseline of writing (the equals sign before the function name).

PRACTICE 15A

LINKED EXPRESSIONS

Instructions: Keep the following points in mind as you select runover sites. (1) Before dividing an expression, try placing the switch indicators on a different line. (2) Before dividing an expression, try placing the identifier on a different line. (3) If the entire expression will not fit on the line, divide before a comparison sign on the baseline.

A. Solve the linear inequalities.

 i. $(x+\frac{10}{3})(x+\frac{19}{3}) > (3x+\frac{46}{3})(\frac{x}{3}+1)$

 ii. $\frac{2x}{3} - 3 > \frac{16x}{21} - \frac{13}{3} - \frac{2x}{15}$

 iii. $(a-1)^2 - (a-7)(a-3) < 2a + 0.8$

B. $2 \times 423 = (2 \times 400) + (2 \times 20) + (2 \times 3) = 800 + 40 + 6 = 846$

Binomial coefficients get their name because they are the *coefficients* in the expansion of a *binomial:*

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

15.4 Step b: Divide Before an Operation Sign on the Baseline

A new line may begin with an operation symbol which is printed on the baseline of writing. It is not necessary to divide at every operation sign. A fraction line is not considered to be an operation sign in this regard.

Keep these points in mind as you study the examples in this section.

- Determine whether the expression can fit on one line using tactics given in Section 15.1.
- If the math will not fit, use as much of the line as possible before dividing.
- The operation sign at which the division is made must be on the baseline of writing.
- The print copy may divide after an operation sign, but the braille transcription follows Nemeth rules and divides *before* the operation symbol. (Example 15-16)
- Transition to a runover line does not take the place of a necessary baseline indicator. If an operation sign follows a superscript, begin the new line with the baseline indicator. (Example 15-10) Similarly, if an operation sign follows a subscript that has a subscript indicator, begin the new line with the baseline indicator. If the subscript does not use a subscript indicator, there will be no subsequent baseline indicator. (Example 15-15) Review Lesson 6 regarding nonuse of the subscript indicator.
- If the new line begins with a minus symbol and a numeral, or with a minus symbol and a decimal point and a numeral, a numeric indicator is required. (Example 15-9)
- Unless unavoidable, an operation sign within a mathematical unit is not a suitable division site. (Examples 15-11 and 15-12)
- In a linked expression, if either anchor or link is divided, further rules apply. See Section 15.4.2.

Example 15-8

THE EXPRESSION REQUIRES 46 CELLS

(embedded)

$1\frac{5}{6}+9\frac{1}{2}+3\frac{1}{12}+2\frac{1}{4}+3\frac{1}{2}$

⠼⠂⠸⠹⠢⠸⠌⠖⠸⠼⠬⠔⠸⠹⠂⠸⠌⠆⠸⠼⠬⠒⠸⠹⠂⠸⠌⠂⠆⠸⠼⠬⠆⠸⠹⠂⠸⠌⠲⠸⠼
⠬⠒⠸⠹⠂⠸⠌⠆⠸⠼

Line 1: The first four terms fit on this line. The expression is divided before the last plus sign.
Line 2 (runover line): A numeral preceded by a plus symbol does not require a numeric indicator.

Example 15-9

THE EXPRESSION REQUIRES 43 CELLS

(displayed)

$$2x^4 + 14x^3 - 15y^3 + 5x^2 - 4y^2 + 16x - 10y + 31$$

⠀⠀⠼⠆⠭⠘⠲⠐⠬⠂⠲⠭⠘⠒⠐⠤⠂⠢⠽⠘⠒⠐⠬⠢⠭⠘⠆⠐⠤⠲⠽⠘⠆⠐⠬⠂⠖⠭⠀⠀

⠀⠀⠀⠀⠤⠼⠂⠴⠽⠬⠒⠂

Line 1: The displayed expression begins in cell 3. All but the last two terms fit on this line.
Line 2: The runover line begins in cell 5 with an operation symbol. A numeral preceded by a minus symbol requires a numeric indicator when the minus symbol begins a braille line.

Example 15-10

THE EXPRESSION REQUIRES 43 CELLS

(displayed)

$$49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2$$

⠀⠀⠼⠲⠔⠭⠘⠶⠐⠽⠘⠖⠐⠤⠖⠒⠭⠘⠖⠐⠽⠘⠲⠐⠬⠢⠖⠭⠘⠢⠐⠽⠘⠢⠀⠀⠀⠀⠀⠀

⠀⠀⠀⠀⠐⠬⠖⠲⠭⠘⠲⠐⠽⠘⠆

Line 1: The displayed expression begins in cell 3. All but the last term fits on this line.
Line 2: The runover line begins with the baseline indicator which precedes the operation symbol (the plus symbol).

15.4.1 A Sign of Operation within a Mathematical Unit

Unless unavoidable, an operation sign within a mathematical unit is not a suitable division site. (See the list of mathematical units in Section 15.2.)

Example 15-11

THE EXPRESSION REQUIRES 42 CELLS

(embedded)

Find a solution, using PEMDAS: $(2^2 + 3^3) - 2^5 \div 4 + (22 \div 2 - 2 \cdot 5) + (14 - 6 \div 6)$

1 ⠀⠀⠠⠋⠔⠙⠀⠁⠀⠎⠕⠇⠥⠰⠝⠂⠀⠥⠎⠬⠀⠠⠠⠏⠑⠍⠙⠁⠎⠒⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

2 ⠸⠩⠀⠷⠆⠘⠆⠐⠬⠒⠘⠒⠐⠾⠤⠆⠘⠢⠐⠨⠌⠲⠬⠷⠆⠆⠨⠌⠆⠤⠆⠡⠢⠾⠀⠀⠀⠀⠀⠀

3 ⠬⠷⠂⠲⠤⠖⠨⠌⠖⠾⠀⠸⠱

Line 1: A grouped expression is a mathematical unit. Although the first addend will fit on line 1, that plus sign is within a grouped expression and so is not a suitable division site.
Line 2: The opening switch is placed on the same line as the beginning of the embedded mathematical expression.
Line 3: The operation sign outside of the grouped expression is the preferred division site. Line 3 begins with a plus symbol that is situated between two groupings.

Example 15-12

THE EXPRESSION REQUIRES 39 CELLS

(displayed)

$$\frac{1}{y^2-6y+8}+\frac{1}{y^2-16}-\frac{5}{y^2+2y-8}$$

```
⠀⠀⠹⠂⠌⠽⠘⠆⠐⠤⠖⠽⠬⠦⠼⠬⠹⠂⠌⠽⠘⠆⠐⠤⠂⠖⠼⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
    ⠤⠹⠢⠌⠽⠘⠆⠐⠬⠆⠽⠤⠦⠼
```

Line 1: Starting in cell 3, the entire math expression will not fit on one line.
Lines 1-2: A fraction is a mathematical unit. Each fraction is kept intact on one line. The operation signs within the denominators are not suitable division sites.
Line 2: The operation sign on the baseline of writing is the preferred division site. Line 2 begins with the minus symbol before the last fraction.

15.4.2 **Further Considerations Within a Linked Expression.** Review the bullet points in Section 15.3 (step a). If an anchor or a link will not fit on the line and must be divided, further rules apply.

i. **A linked expression with only one link.** If a division must be made *within* the anchor or its link, a division must still be made before the link. In other words, step a is applied first, then step b. (Examples 15-13 and 15-14.)

ii. **A linked expression with two or more links.** In a linked expression with more than one link, if a division must be made *within* the anchor or any link, a division must still be made before each link. The resulting transcription maintains an orderly representation of mathematical units. (Example 15-16)

Example 15-13

ANCHOR: 10 CELLS; LINK: 49 CELLS

(displayed)

$$20\frac{1}{6}=1\frac{5}{6}+9\frac{1}{2}+3\frac{1}{12}+2\frac{1}{4}+3\frac{1}{2}$$

```
1   ⠀⠀⠼⠆⠴⠸⠹⠂⠌⠖⠸⠼⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
2       ⠨⠅ ⠼⠂⠸⠹⠢⠌⠖⠸⠼⠬⠔⠸⠹⠂⠌⠆⠸⠼⠬⠒⠸⠹⠂⠌⠂⠆⠸⠼⠀⠀⠀⠀⠀
3       ⠬⠆⠸⠹⠂⠌⠲⠸⠼⠬⠒⠸⠹⠂⠌⠆⠸⠼
```

Line 1: The anchor is on line 1.
Line 2: The runover line begins with the comparison symbol—the equals sign (step a). The link does not fit entirely on line 2.
Line 3: The second runover line begins with an operation symbol—a plus sign (step b).

Example 15-14

ANCHOR: 48 CELLS; LINK: 5 CELLS

(itemized)

5. $144\ \mathrm{ft}^2 + 17\ \mathrm{ft}^2 + 112\ \mathrm{ft}^2 + 15\ \mathrm{ft}^2 - 131\ \mathrm{ft}^2 = N$

```
1 ⠼⠢⠸⠲ ⠼⠂⠲⠲ ⠋⠞⠘⠆ ⠬⠂⠶ ⠋⠞⠘⠆ ⠬⠂⠂⠆ ⠋⠞⠘⠆
2   ⠬⠂⠢ ⠋⠞⠘⠆ ⠤⠼⠂⠒⠂ ⠋⠞⠘⠆
3   ⠨⠅ ⠠⠝
```

Line 1: After checking that the anchor will not fit entirely on line 2, it begins here.
Line 2: The anchor is divided before the plus sign of the fourth term (step b).
Line 3: The linked expression is divided before the equals sign (step a). The link must not be placed on line 2 even though it will fit because step b has been applied—that is, because the anchor has been divided.

Example 15-15

ANCHOR: 34 CELLS; LINK: 37 CELLS

(itemized)

1. Consider the following chemical equations.

   a. $\mathrm{CH_3CH_2CH_2COONa+CH_3OH} \rightarrow \mathrm{CH_3CH_2CH_2COOCH_3+NaOH}$

   b. …

```
1 ⠼⠁⠲ ⠠⠒⠎⠊⠙⠻ ⠮ ⠋⠕⠇⠇⠪⠬ ⠡⠑⠍⠊⠉⠁⠇ ⠑⠟⠥⠁⠰⠝⠎⠲ ⠸⠩
2   ⠰⠁⠸⠲
3     ⠠⠉⠠⠓⠒⠠⠉⠠⠓⠆⠠⠉⠠⠓⠆⠠⠉⠠⠕⠠⠕⠠⠝⠁⠬⠠⠉⠠⠓⠒⠠⠕⠠⠓
4     ⠫⠕ ⠠⠉⠠⠓⠒⠠⠉⠠⠓⠆⠠⠉⠠⠓⠆⠠⠉⠠⠕⠠⠕⠠⠉⠠⠓⠒
5     ⠬⠠⠝⠁⠠⠕⠠⠓
6   ⠰⠃⠸⠲ ⠄⠄⠄ ⠸⠱
```

Lines 2-3: The anchor does not fit on the line with the identifier, but it will fit entirely on the next line. It begins in the runover position for subdivisions (cell 5).
Line 4: Line 4 begins with the comparison symbol (right-pointing arrow, step a). The link will not fit entirely on this line. It is divided before the operation sign (plus sign, step b).
Line 5: Because the subscript (3) at the end of line 4 does not require a subscript indicator, there is no baseline indicator needed before the plus symbol that begins on line 5.

Example 15-16

ANCHOR: 46 CELLS; LINKS: 15, 14, 13 CELLS

(embedded)

Show how Margaret solved this problem by using fraction cubes. $1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2} = 18\frac{26}{12} = 20\frac{2}{12} = 20\frac{1}{6}$. Can you express the result as a decimal?

```
1     ⠠⠩⠪ ⠓⠪ ⠠⠍⠜⠛⠜⠑⠞ ⠎⠕⠇⠧⠫ ⠹ ⠏⠗⠕⠃⠇⠑⠍ ⠃⠽ ⠥⠎⠬
2   ⠋⠗⠁⠉⠰⠝ ⠉⠥⠃⠑⠎⠲ ⠸⠩ ⠼⠂⠸⠹⠢⠌⠖⠸⠼⠬⠔⠸⠹⠂⠌⠆⠸⠼
3   ⠬⠒⠸⠹⠂⠌⠂⠆⠸⠼⠬⠆⠸⠹⠂⠌⠲⠸⠼⠬⠒⠸⠹⠂⠌⠆⠸⠼
4   ⠨⠅ ⠼⠂⠦⠸⠹⠆⠖⠌⠂⠆⠸⠼
5   ⠨⠅ ⠼⠆⠴⠸⠹⠆⠌⠂⠆⠸⠼
6   ⠨⠅ ⠼⠆⠴⠸⠹⠂⠌⠖⠸⠼ ⠸⠱⠲ ⠠⠉ ⠽ ⠑⠭⠏⠗⠑⠎⠎ ⠮ ⠗⠑⠎⠥⠇⠞
7   ⠵ ⠁ ⠙⠑⠉⠊⠍⠁⠇⠦
```

Line 2: The anchor will not fit on one line. It begins on line 2 and is divided before a plus sign (step b).
Line 3: The anchor is completed on this line. Division is made before the first link (step a).
Lines 4-6: Each link begins on a new line because the anchor is divided.
Line 6: The paragraph continues following the completion of the last link.

PRACTICE 15B

Instructions: In addition to the tactics outlined with Practice 15A, keep the following point in mind as you select runover sites. Apply step a (divide before a comparison sign) before applying step b (divide before an operation sign).

A. $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 = 490°$

B. $2\frac{3}{4}$ yd $+ 1\frac{3}{4}$ yd $+ \frac{3}{4}$ yd $= 5\frac{1}{4}$ yd

C. $\sqrt{(x+a^2)+(y+a^2)} - \sqrt{(x-a^2)+(y-a^2)} = \pm 2a$

D. Sommer's routine can be represented by the following expression.

Routine S: $5 + 3 + 4(1 + (-1)) + (-3) + (-5) + 3\big(5 + (-2)\big) + 1$

Draw a simple diagram to represent *Routine S*.

15.5 Step c: Divide Before a Mathematical Unit

When steps a and b do not apply or when further division is necessary, step c is summoned. By seeing an expression as a series of mathematical units, the transcriber can make wise decisions when a long expression must be divided. Prudent division sites allow the reader to mentally reassemble the expression in an orderly manner.

15.5.1 **A Fraction is a Unit.** A fraction as a whole is a mathematical unit. Within a fraction, the numerator and the denominator are each a unit as well. If an expression requires division when a fraction is encountered, follow these guidelines.

- If the fraction will fit on the line, do not divide it.
- If the entire fraction will not fit on one line, divide before the fraction line. (Example 15-17) If a baseline indicator is required before the fraction line, divide the expression before the baseline indicator. (Example 15-18)
- If the numerator or denominator requires division, apply further division strategies with that unit. Division must also be made before the fraction line if numerator or denominator are divided. (Example 15-18)
- If the fraction is part of a mixed number, do not separate the fractional part from its whole number. Review the topic of mixed numbers in Lesson 8.

Example 15-17

NUMERATOR: 27 CELLS; DENOMINATOR: 15 CELLS

(displayed)

$$\frac{93{,}000 \times 0.0006 \times 2.0 \times 10.56}{32 \times 1.257 \times 10}$$

⠀⠀⠹⠔⠒⠠⠴⠴⠴⠈⠡⠴⠨⠴⠴⠴⠖⠈⠡⠆⠨⠴⠈⠡⠂⠴⠨⠢⠖⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠌⠒⠆⠈⠡⠂⠨⠆⠢⠶⠈⠡⠂⠴⠼

The fraction (42 cells) will not fit on one line. Division is made before the fraction line. This allows both the numerator and the denominator to remain intact.

Example 15-18

NUMERATOR: 49 CELLS; DENOMINATOR: 4 CELLS

(displayed)

$$\frac{e^{1/n}+e^{2/n}+e^{3/n}+\ \dots+e^{(n-1)/n}+e^{n/n}}{n}$$

1 ⠀⠀⠹⠑⠘⠂⠸⠌⠝⠐⠬⠑⠘⠆⠸⠌⠝⠐⠬⠑⠘⠒⠸⠌⠝⠐⠬ ⠄⠄⠄
2 ⠀⠀⠀⠀⠬⠑⠘⠷⠝⠤⠂⠾⠸⠌⠝⠐⠬⠑⠘⠝⠸⠌⠝
3 ⠀⠀⠀⠀⠐⠌⠝⠼

Looking at the fraction as a whole, the first division site is before the fraction line. This will become line 3.
Lines 1-2: The numerator will not fit on one line. It is divided before a sign of operation that is situated on the baseline within the numerator (step b within a unit).
Line 3: Even though the denominator will fit on line 2, division is required before the fraction line. A baseline indicator is needed before this fraction line because the last item in the numerator is a superscript. The baseline indicator is not separated from the fraction line.

15.5.2 **A Grouped Expression is a Unit.** When a series of unspaced grouped expressions does not fit on the line, division may be made between groupings. The new line begins with the left grouping symbol of the next factor. (Example 15-19)

If a grouped expression will not fit on one line, apply further division strategies within the grouping. (Example 15-20)

Example 15-19

THE EXPRESSION REQUIRES 44 CELLS

(embedded)

Multiply these six terms: $(x+5)(2x-1)(x^2+2x+1)\ (x^2+3x+4)(4-x)(5x+7)$.

1 ⠀⠀⠠⠍⠥⠇⠞⠊⠏⠇⠽ ⠘⠮ ⠎⠊⠭ ⠞⠻⠍⠎⠒ ⠸⠩ ⠷⠭⠬⠢⠾⠷⠆⠭⠤⠂⠾
2 ⠷⠭⠘⠆⠐⠬⠆⠭⠬⠂⠾⠷⠭⠘⠆⠐⠬⠒⠭⠬⠲⠾⠷⠲⠤⠭⠾⠷⠢⠭⠬⠶⠾ ⠸⠱⠲

The first two factors fit on line 1. Division is made before the third factor.
Line 2 begins with the left parenthesis.

Example 15-20

THE EXPRESSION REQUIRES 55 CELLS

(displayed)

$$(49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2) \div 7x^3y^2$$

```
1   ⠀⠀⠷⠲⠔⠭⠘⠶⠐⠽⠘⠖⠐⠤⠖⠒⠭⠘⠖⠐⠽⠘⠲⠐⠬⠢⠖⠭⠘⠢⠐⠽⠘⠢⠀⠀⠀⠀⠀⠀
2       ⠐⠬⠖⠲⠭⠘⠲⠐⠽⠘⠆⠐⠾
3       ⠨⠌⠶⠭⠘⠒⠐⠽⠘⠆
```

Working through the steps,
Step a does not apply because there is no comparison sign.
Step b (operation sign on the baseline): The expression is first divided before the operation sign that is <u>outside</u> of the grouped expression (divided-by sign). This will become line 3.
Step c (mathematical unit): The grouped expression will not fit on one line. The first three terms fit on line 1. Division within the grouping is made before plus sign of the fourth term (step b within a unit). The baseline indicator associated with that plus symbol begins line 2.
Line 3: Although the rest of the problem will fit on line 2, division is made before the division sign (step b).

15.5.3 **A Base and its Exponent are a Unit; a Subscript and its Related Item are a Unit.**

If the unit must be divided, divide the longer part—the base, or the superscript or subscript. (See Examples 15-21 and 15-26) At other times it makes more sense to begin the new line with the level indicator. (Example 15-22) When transition to a new braille line is made within a superscript or subscript, the level in effect continues just as it would if the expression were not divided.

Example 15-21

THE EXPRESSION REQUIRES 48 CELLS

(embedded)

$x^{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\dots+\frac{1}{n}}$

```
⠭⠘⠂⠬⠹⠂⠌⠆⠼⠬⠹⠂⠌⠒⠼⠬⠹⠂⠌⠲⠼⠬⠹⠂⠌⠢⠼⠬⠹⠂⠌⠖⠼⠬ ⠄⠄⠄
⠬⠹⠂⠌⠝⠼
```

The superscript (47 cells) will not fit on one line. It begins on the same line as the base and is divided before the last term (step b within a unit). The superscript level initiated on the first line (in cell 2) continues on the next line without need for restatement. (Review Section 6.12.4 in Lesson 6 regarding an ellipsis in the superscript position.)

Example 15-22

THE EXPRESSION REQUIRES 48 CELLS

(embedded)

$$\frac{1}{2}\left[\frac{1}{2} sin\, 2u - u\right]_{A(u=0)}^{P(u=u)}$$

⠹⠂⠌⠆⠼⠈⠷⠹⠂⠌⠆⠼⠎⠊⠝ ⠼⠆⠥⠤⠥⠈⠾⠰⠠⠁⠷⠥ ⠰⠨⠅ ⠼⠴⠾
⠘⠠⠏⠷⠥ ⠘⠨⠅ ⠥⠾

The best option is to keep grouped expression between brackets undivided (18 cells). The subscript is transcribed first, according to Nemeth rules for simultaneous superscripts and subscripts (Lesson 6). The subscript (13 cells) fits on the first line, but the superscript (12 cells) does not. The superscript begins the runover line, starting with the superscript indicator.

15.5.4 **Further Considerations Within a Linked Expression.** Review the bullet points in Sections 15.3 (step a) and 15.4 (step b). If a step c division must be made within an anchor or within a link, work your way through these steps.

- First, divide before the link, starting the runover line with the comparison symbol (step a). If there is more than one link, start a new line with each link.
- Within the anchor or link that will not fit on the line, divide before an operation sign on the baseline, if present (step b).
- Finally, divide before a mathematical unit (step c).

Example 15-23

ANCHOR: 44 CELLS; LINK: 5 CELLS

(embedded linked expression)

In this model, $\frac{\text{Fixed Cost}}{\text{Unit Revenue}-\text{Marginal Cost}} = P$

1 ⠠⠔ ⠹ ⠍⠕⠙⠑⠇⠂ ⠸⠩ ⠹⠠⠋⠊⠭⠑⠙ ⠠⠉⠕⠎⠞
2 ⠌⠠⠥⠝⠊⠞ ⠠⠗⠑⠧⠑⠝⠥⠑⠤⠠⠍⠁⠗⠛⠊⠝⠁⠇ ⠠⠉⠕⠎⠞⠼
3 ⠨⠅ ⠠⠏ ⠸⠱

The anchor (a fraction) needs to be divided. Working through the steps,

Step a (comparison sign on the baseline): The first division is made before the link. This will become line 3 which begins with the equals symbol.

Step b does not apply because the operation sign (minus sign) is not on the baseline.

Step c (mathematical unit): Since the fraction will not fit on one line, division is made before the fraction line. The numerator fits on line 1. The denominator fits on line 2, beginning with the fraction line and ending with the closing fraction indicator.

Line 3: Even though the link will fit on line 2, division before the comparison sign is required (step a).

Example 15-24

ANCHOR: 1 CELL; LINK: 48 CELLS

(displayed linked expression)

$$x > \frac{93{,}000 \times 0.0006 \times 2.0 \times 10.56}{32 \times 1.257 \times 10} + 76$$

1 ⠀⠀⠭
2 ⠀⠀⠀⠀⠨⠂ ⠹⠔⠒⠠⠴⠴⠴⠈⠡⠴⠨⠴⠴⠴⠖⠈⠡⠆⠨⠴⠈⠡⠂⠴⠨⠢⠖⠀⠀⠀⠀⠀⠀
3   ⠌⠒⠆⠈⠡⠂⠨⠆⠢⠶⠈⠡⠂⠴⠼⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
4   ⠬⠶⠖

The link needs to be divided. Working through the steps,
Step a (comparison sign on the baseline): The first division is made before the link. Line 2 begins with the greater than symbol.
Step b (operation sign on the baseline): The link is first divided before the plus sign. This will become line 4.
Step c (mathematical unit): The fraction must be divided. A division is made before the fraction line.
Line 4: Even though the second term will fit on line 3, division before the operation sign is required (step b).

Example 15-25

ANCHOR: 1 CELL; LINK: 43 CELLS

(displayed linked expression)

$$y = \left(-8,\ -4,\ \tfrac{1}{2},\ 3,\ 8,\ 10,\ 20\tfrac{3}{4},\ 100\right)$$

1 ⠀⠀⠽
2 ⠀⠀⠀⠀⠨⠅ ⠷⠤⠦⠠ ⠤⠲⠠ ⠹⠂⠌⠆⠼⠠ ⠒⠠ ⠦⠠ ⠂⠴⠠⠀⠀⠀⠀⠀⠀⠀⠀
3   ⠆⠴⠸⠹⠒⠌⠲⠸⠼⠠ ⠂⠴⠴⠾

The link needs to be divided. Working through the steps,
Step a (comparison sign on the baseline): The first division is made before the link. Line 2 begins with the equals symbol.
Step b does not apply. There are no operation signs in this expression.
Step c (mathematical unit): The grouped expression is an enclosed list. Division is made <u>*after*</u> *a comma. (Lesson 4)*
Line 1: Even though there is room for part of the link on line 1, division before the comparison sign is required (step a).

Example 15-26

ANCHOR: 34 CELLS; LINK: 35 CELLS

(displayed linked expression)

1. Using the product of powers property to simplify the first term,

$$(6^2 6^3 x^{\frac{1}{2}-2} y^{\frac{1}{3}-3} + z)^3 = (6^{2+3} x^{\frac{1}{2}-2} y^{\frac{1}{3}-3} + z)^3.$$

```
1   ⠼⠁⠲ ⠠⠥⠎⠬ ⠮ ⠏⠗⠕⠙⠥⠉⠞ ⠷ ⠏⠪⠻⠎ ⠏⠗⠕⠏⠻⠞⠽ ⠞⠕
2     ⠎⠊⠍⠏⠇⠊⠋⠽ ⠮ ⠋⠌ ⠞⠻⠍⠂ ⠸⠩
3       ⠷⠖⠘⠆⠐⠖⠘⠒⠐⠭⠘⠹⠂⠌⠆⠼⠤⠆⠐⠽⠘⠹⠂⠌⠒⠼⠤⠒⠐⠬⠵⠾⠘⠒
4         ⠨⠅ ⠷⠖⠘⠆⠬⠒⠐⠭⠘⠹⠂⠌⠆⠼⠤⠆⠐⠽⠘⠹⠂⠌⠒⠼⠤⠒
5         ⠐⠬⠵⠾⠘⠒ ⠸⠱⠲
```

Line 3: The anchor fits on one line. Division is made before the comparison sign (step a).

Line 4: The link consists of one mathematical unit: a base and its exponent. The base consists of two addends. The link needs to be divided. Division is made within the base, before the plus line printed on the baseline. The exponent is undivided from its base, although this means dividing a grouped expression.

Example 15-27

ANCHOR: 1 CELL; LINK: 37 CELLS

(displayed linked expression)

$$\mathrm{B} = \frac{7!}{5!(7-5)!}(x^2)^{7-5}\left(-\frac{y}{2}\right)^5$$

```
1   ⠀⠀⠀⠠⠃
2   ⠀⠀⠀⠀⠨⠅ ⠹⠶⠯⠌⠢⠯⠷⠶⠤⠢⠾⠯⠼⠷⠭⠘⠆⠐⠾⠘⠶⠤⠢⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
3       ⠐⠷⠤⠹⠽⠌⠆⠼⠾⠘⠢
```

Line 2: The first division is made before the comparison sign (step a). The link consists of three factors and will not fit on one line. Division is made before the third factor.

Line 3: The base and its exponent are a unit and so are not divided. Because the left parenthesis follows a superscript, the necessary baseline indicator begins the runover line.

15.5.5 Divide Before a Baseline Indicator

As seen in many of the examples, when a division site has an associated baseline indicator, the new line begins with the indicator.

PRACTICE 15C

Instructions: Note that the ellipsis in item c. is on the baseline of writing. Review Section 8.22.4 in Lesson 8 regarding the layout of problem 2.

1. Compute and/or simplify.

a. $\frac{\left(\frac{3}{2}\right)\times\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{1\times 2\times 3}$

b. $\frac{\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\frac{\mathrm{d}y}{\mathrm{d}t}}{(\mathrm{d}x/\mathrm{d}t)^3}$

c. $(\pm)a_{1i_1}a_{2i_2}a_{3i_3}a_{4i_4}\ldots a_{ni_n}$

2. $$\frac{x^2+2}{x(2x^2+1)^2} = \frac{A}{x}+\frac{Bx+C}{2x^2+1}+\frac{Dx+E}{(2x^2+1\,)^2}$$

$$= \frac{A\left(4x^4+4x^2+1\right)+B\left(2x^4+x^2\right)+C\left(2x^3+x\right)+Dx^2+Ex}{x(2x^2+1)^2}$$

PRACTICE 15D

1. Multiply these four polynomials: $(4x^9y)(7x^8y^2)(5x^7y^3)(6x^6y^4)$.

2. $\pm\left(\frac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}{3}\right)^{\frac{3}{2}}$

3. $(16x^4+8x^3y+4x^2y^2+2xy^3+y^4)(2x-y)=32x^5-y^5$

15.6 Step d: Divide After a Termination Indicator

If a suitable division site cannot be found and if a termination indicator is present, begin a new line after the termination indicator.

15.6.1 **A Radical Expression is a Unit.** Division may be made after the termination indicator that ends a radical expression. If the entire radical expression will not fit on the line, apply division strategies to the radicand.

Example 15-28

THE EXPRESSION REQUIRES 41 CELLS

(displayed)

$$\sqrt[5]{32a^5b^5c^5d^5}\ \sqrt[5]{2abc^2d^3e^2}$$

```
⠀⠀⠣⠢⠜⠒⠆⠁⠘⠢⠐⠃⠘⠢⠐⠉⠘⠢⠐⠙⠘⠢⠐⠻⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
    ⠣⠢⠜⠆⠁⠃⠉⠘⠆⠐⠙⠘⠒⠐⠑⠘⠆⠐⠻
```

Division is made <u>after</u> the termination indicator of the first radical expression.

15.6.2 **A Modified Expression is a Unit.** Division may be made after the termination indicator that ends an expression which is modified according to the five-step rule of modification. (Lesson 12) If the entire modified expression will not fit on the line it is acceptable to divide before the directly over or the directly under indicator. That strategy will be illustrated in the next section with Example 15-34.

Example 15-29

ANCHOR: 2 CELLS; LINK: 55 CELLS

(displayed)

$$V = \lim_{\Delta x \to 0} \sum_{i=1}^{n} \pi \left[f(x_i) \right]^2 \Delta x$$

```
1   ⠀⠀⠠⠧⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
2   ⠀⠀⠀⠀⠨⠅ ⠐⠇⠊⠍⠩⠨⠠⠙⠭ ⠫⠕ ⠼⠴⠻ ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠂⠣⠝⠻
3       ⠨⠏⠈⠷⠋⠷⠭⠰⠊⠐⠾⠈⠾⠘⠆⠐⠨⠠⠙⠭
```

Line 2: The first division is made before the link (step a). The link does not fit on one line.
Division is made <u>after</u> the termination indicator which ends the second modified expression.

PRACTICE 15E

Note: In item 2, the expression is displayed.

1. $\sqrt[3]{x^1}\ \sqrt[6]{x^2}\ \sqrt[12]{x^3}\ \sqrt[24]{x^4}\ \sqrt[48]{x^5}\ \ldots$
2. The general solution for one root of the cubic equation is

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\ .$$

15.7 Function Notation, Integral Notation, Sigma Notation, and Pi Notation

This lesson concludes with some rather complicated looking notation which is commonly encountered in the study of calculus. Frequently, the transcriber needs to choose between which mathematical unit to keep intact and which unit to divide. There is often more than one valid layout. As with any formatting choice, consistency in your treatment of runovers in complicated mathematical expressions will give your reader the advantage of knowing what to expect. By keeping in mind the strategies presented in this lesson, your transcription will be well done.

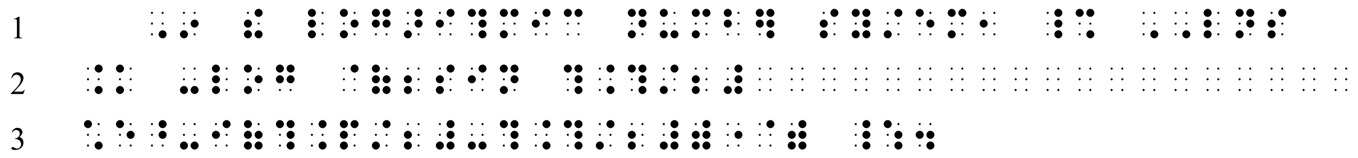
15.7.1 **A Function and its Argument are a Unit.** In Lesson 14 it was demonstrated that a function name and the sign which follows it (known as the *argument*) are regarded as a single mathematical item and therefore should not be divided between braille lines. If the entire expression will not fit on the line, apply division strategies to the argument. Sometimes this will mean dividing within a mathematical unit.

Example 15-30

ANCHOR: 5 CELLS; LINK: 44 CELLS

(embedded)

In the logarithmic number system, LNS $= -\log\left[2\sin\frac{\theta}{2}\cdot e^{-i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)}\right]$.



Line 1: The anchor fits on line 1. Division is made before the link (step a). (LNS is the abbreviation for "logarithmic number system" and so uses the double capitalization indicator.)

Lines 2-3: Even though the grouped expression will fit all together on line 3, division is made within the grouping in order to begin the argument on the same line as the abbreviated function name (log). Line 3 begins with the multiplication dot (step b within a unit).

PRACTICE 15F

1. Daylight …

   a. The number of minutes of daylight for any location at 60° N latitude is modeled by the function below, where $d =$ day of the year.

$$m(d) = 390\sin\left[\frac{2\pi}{365.25}(d-80)\right] + 738$$

15.7.2 **Integral Notation.** To aid in our discussion, here are the names of the parts in integral notation.

$$\int_a^b f(x)\,dx$$

a and b are the *limits.* $f(x)$ is the *integrand.* dx is the *differential.*

Division strategies are illustrated showing the same example using two styles of integral notation—printing the limits as a superscript and subscript (Example 15-31) and printing the limits as modifiers (Example 15-32).

Example 15-31

THE EXPRESSION REQUIRES 40 CELLS

$$\int_{-\infty}^{\infty}\frac{\frac{a(x)}{b(x)}}{\frac{c(x)}{f(x)}}\,dx$$

(embedded)

⠮⠰⠤⠠⠿⠘⠠⠿⠐⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

The 40-cell expression will fit on one line when embedded within a 3-1 paragraph. (See Section 13.1.8.a in Lesson 13 regarding simultaneous superscripts and subscripts with the slanted integral sign.)

Here is an analysis of the parts.

The *limits* are a subscript and a superscript. ⠰⠤⠠⠿⠘⠠⠿

The *integrand* is a complex fraction.

⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼

The *differential* is dx. ⠙⠭

(displayed)

Using displayed margins starting in cell 5, the expression will no longer fit on one line.

⠀⠀⠀⠀⠮⠰⠤⠠⠿⠘⠠⠿

⠐⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭⠀⠀⠀

By dividing before the fraction, the fraction is kept intact. The baseline indicator associated with the opening complex fraction indicator begins the runover line.

Example 15-32

THE EXPRESSION REQUIRES 40 CELLS

$$\int_{-\infty}^{\infty}\frac{\frac{a(x)}{b(x)}}{\frac{c(x)}{f(x)}}dx$$

(embedded)

⠐⠮⠩⠤⠠⠿⠣⠠⠿⠻⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

The 40-cell expression will fit on one line when embedded within a 3-1 paragraph

Here is an analysis of the parts.

The *limits* are modifiers. ⠩⠤⠠⠿⠣⠠⠿

The *integrand* is a complex fraction.

⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼

The *differential* is dx. ⠙⠭

(displayed)

Using displayed margins starting in cell 3, the expression will no longer fit on one line.

⠀⠀⠐⠮⠩⠤⠠⠿⠣⠠⠿⠻

⠀⠀⠀⠀⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

Dividing after the termination indicator of the modified integral works well here (step d).

PRACTICE 15G

Note: The unusual letter in the last numerator is a lowercase Greek zeta.

The solutions involving the direct functions

$$\int \frac{\log^2(z+1)}{z}\,\mathrm{d}z = \log(-z)\log^2(z+1) + 2\mathrm{Li}_2(z+1)\log(z+1) - 2\mathrm{Li}_3(z+1)$$

and

$$\int_0^\infty \frac{\log(t+1)\log\left(1+\frac{1}{t^2}\right)}{t}\,\mathrm{d}t = C\pi - \frac{3\zeta(3)}{8}$$

can be found in Chapter 7.

15.7.3 **Sigma and Pi Notation.** Step d works nicely with these long expressions.

Example 15-33

ANCHOR: 35 CELLS; LINK: 38 CELLS

$$\prod_{i=1}^{2}\prod_{j=4}^{6}(3ij) = \prod_{i=1}^{2}\big((3i\cdot 4)(3i\cdot 5)(3i\cdot 6)\big)$$

```
1   ⠀⠀⠐⠨⠠⠏⠩⠊⠀⠨⠅⠀⠼⠂⠣⠆⠻⠐⠨⠠⠏⠩⠚⠀⠨⠅⠀⠼⠲⠣⠖⠻⠷⠒⠊⠚⠾
2   ⠀⠀⠀⠀⠨⠅⠀⠐⠨⠠⠏⠩⠊⠀⠨⠅⠀⠼⠂⠣⠆⠻
3   ⠀⠀⠀⠀⠷⠷⠒⠊⠡⠲⠾⠷⠒⠊⠡⠢⠾⠷⠒⠊⠡⠖⠾⠾
```

Line 1: The anchor fits on one line. Division is made before the link (step a).
Line 2: The link must be divided. Division is made after the termination indicator.
Line 3: The grouped expression fits all together on one line.

Example 15-34

ANCHOR: 17 CELLS; LINK: 69 CELLS

$$d(\vec{x},\vec{y}) =$$

$$\sum_{\substack{Z_{xy}\in\vec{Z}_{xy}\\ \forall x\in\vec{x}\\ \forall y\in\vec{y}}} f(Z_{xy})$$

```
1   ⠀⠀⠙⠷⠐⠭⠣⠫⠕⠻⠠⠀⠐⠽⠣⠫⠕⠻⠾
2   ⠀⠀⠀⠀⠨⠅⠀⠐⠨⠠⠎⠩⠠⠵⠰⠭⠽⠀⠈⠑⠀⠐⠠⠵⠣⠫⠕⠻⠰⠭⠽
3   ⠀⠀⠀⠀⠐⠩⠩⠈⠯⠭⠀⠈⠑⠀⠐⠭⠣⠫⠕⠻⠩⠩⠩⠈⠯⠽⠀⠈⠑⠀⠐⠽⠣⠫⠕⠻⠻
4   ⠀⠀⠀⠀⠋⠷⠠⠵⠰⠭⠽⠐⠾
```

Line 1: The anchor fits on one line. Division is made before the link (step a) despite how it is printed.
Line 2: The link begins in cell 5. The entire modified expression will not fit on the line. Since all three modifiers will not fit on one line (53 cells), division is made before the second order directly under indicator.
Line 3: The baseline indicator associated with the directly under indicator begins the line. There is not enough room to complete the link. Division is made after the termination indicator of the modification.
Line 4: The argument fits all together on one line.

PRACTICE 15H

Instructions: Transcribe this expression as if it were embedded within narrative, beginning with an opening Nemeth Code indicator in cell 1. Show two different ways to divide this expression.

$$\sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n} \frac{z^n}{n!}$$

15.7.4 **Notation Combinations**

Example 15-35

ANCHOR: 12 CELLS; LINK: 44 CELLS

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\Delta x\cdot f(x_i)$$

Print Observations: The link will not fit on one line. The mathematical units in the link are an abbreviated function name (lim) and its argument (the Sigma notation), modified expressions, and factors in multiplication.

```
1   ⠀⠀⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭
2     ⠨⠅ ⠐⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠂⠣⠝⠻
3     ⠨⠠⠙⠭⠡⠋⠷⠭⠰⠊⠐⠾
```

Line 1: The anchor fits on one line. Division is made before the link (step a).
Line 2: Division is made after the <u>*second*</u> *termination indicator in order to keep the function name and its argument on the same line, although the argument itself must be divided. (Recall from Lesson 14 that, when a function name carries a modifier, the required space after the function name follows the termination of the modifier.)*
Line 3: The two factors fit on this line.

PRACTICE 15I

Instructions: Show two different ways to divide this expression.

1. The rectangle rule for numerical integration (The Midpoint Rule) is inspired by the use of Riemann sums.

$$\frac{b-a}{n}\sum_{i=0}^{n-1} f(a+i\frac{b-a}{n}) \approx \int_a^b f(x)dx$$

SUMMARY

Keep these basic points in mind as you consider the layout of a long mathematical expression. First, determine whether the expression can fit on one line using the tactics given in Section 15.1. If the math must be divided, use as much of the line as possible before dividing. The following division strategies were explored in this lesson.

- When a linked expression will not fit on one line, division is made before the comparison sign that begins the link.
  - When the anchor will not fit on one line, division is made within the anchor.
  - When a link will not fit on one line, division is made within the link.
  - In a linked expression, if either the anchor or any link must be divided, a division must also be made before each link.
- Fractions are kept intact by dividing before an opening fraction indicator. If a fraction must be divided, division is made before the fraction line. A mixed number should not be divided.
- Transition to a runover line does not take the place of a necessary baseline indicator. The baseline indicator will be the first symbol on the new line.
- When a base and its superscript or subscript are too long to fit on a single braille line, division is made either within the base, within the superscript or subscript, or before a change-of-level indicator.
- A function name and its argument should not be divided at the space between them. If the entire expression will not fit on the line, division strategies are applied to the argument.
- Items within grouping signs should not be divided. If consecutive groupings do not fit on the line, a division may be made between groupings.
  - If a grouped expression will not fit on one line, division strategies are applied within the grouping. If the grouped expression is an enclosed list, division is made after a comma.
- Division may be made after a mathematical termination indicator such as termination of a radical expression or termination of a modified expression.
- When an embedded math expression must be divided, it may begin on the current line provided division is made in accordance with the principles defined in this section.
- When a displayed math expression must be divided, all runovers begin two cells to the right of the original display cell unless it is a nested linked expression in which case a second indent level is applied. (See Section 8.22 in Lesson 8.)

Submit Exercise 15 to your instructor.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 15A

```
1   ⠠⠁⠲ ⠠⠎⠕⠇⠧⠑ ⠮ ⠇⠔⠑⠜ ⠔⠑⠟⠥⠁⠇⠊⠞⠊⠑⠎⠲ ⠸⠩
2     ⠰⠊⠸⠲ ⠷⠭⠬⠹⠂⠴⠌⠒⠼⠾⠷⠭⠬⠹⠂⠔⠌⠒⠼⠾
3       ⠨⠂ ⠷⠒⠭⠬⠹⠲⠖⠌⠒⠼⠾⠷⠹⠭⠌⠒⠼⠬⠂⠾
4     ⠰⠊⠊⠸⠲
5       ⠹⠆⠭⠌⠒⠼⠤⠒ ⠨⠂ ⠹⠂⠖⠭⠌⠆⠂⠼⠤⠹⠂⠒⠌⠒⠼⠤⠹⠆⠭⠌⠂⠢⠼
6     ⠰⠊⠊⠊⠸⠲ ⠷⠁⠤⠂⠾⠘⠆⠐⠤⠷⠁⠤⠶⠾⠷⠁⠤⠒⠾ ⠐⠅ ⠼⠆⠁⠬⠴⠨⠦
7   ⠰⠠⠃⠸⠲ ⠼⠆⠈⠡⠲⠆⠒ ⠨⠅ ⠷⠆⠈⠡⠲⠴⠴⠾⠬⠷⠆⠈⠡⠆⠴⠾⠬⠷⠆⠈⠡⠒⠾
8     ⠨⠅ ⠼⠦⠴⠴⠬⠲⠴⠬⠖ ⠨⠅ ⠼⠦⠲⠖ ⠸⠱
9
10    ⠘⠂⠠⠃⠔⠕⠍⠊⠁⠇ ⠘⠂⠉⠕⠑⠖⠊⠉⠊⠢⠞⠎ ⠛⠑⠞ ⠸⠮ ⠐⠝ ⠆⠉
11  ⠮⠽ ⠜⠑ ⠮ ⠨⠂⠉⠕⠑⠖⠊⠉⠊⠢⠞⠎ ⠔ ⠮ ⠑⠭⠏⠁⠝⠨⠝ ⠷ ⠁
12  ⠨⠂⠃⠔⠕⠍⠊⠁⠇⠒ ⠸⠩
13    ⠷⠭⠬⠽⠾⠘⠝
14      ⠨⠅ ⠐⠨⠠⠎⠩⠅ ⠨⠅ ⠼⠴⠣⠝⠻⠷⠝⠩⠅⠾⠭⠘⠅⠐⠽⠘⠝⠤⠅ ⠸⠱⠲
```

Line 2: The anchor fits on the line with the identifier.
Line 3: The link begins in the runover cell (cell 5).
Line 5: This expression will fit on one line if it begins on the line below its identifier.
Line 7: The anchor and the first link fit on the line with the identifier.
Line 8: The remaining two links fit on one line, beginning in the runover cell (cell 3).
Line 9: A blank line precedes the change in format from itemized material to a narrative paragraph.
Line 14: The runover line starts with the comparison symbol that begins the link. The equals sign in the modifier is not a suitable division site because a modifier is a mathematical unit.

PRACTICE 15B

```
1   ⠸⠩
2   ⠰⠠⠁⠸⠲ ⠫⠪ ⠼⠂⠬⠫⠪ ⠼⠆⠬⠫⠪ ⠼⠒⠬⠫⠪ ⠼⠲⠬⠫⠪ ⠼⠢
3     ⠬⠫⠪ ⠼⠖⠬⠫⠪ ⠼⠶
4     ⠨⠅ ⠼⠲⠔⠴⠘⠨⠡
5   ⠰⠠⠃⠸⠲
6     ⠼⠆⠸⠹⠒⠌⠲⠸⠼ ⠽⠙ ⠬⠂⠸⠹⠒⠌⠲⠸⠼ ⠽⠙ ⠬⠹⠒⠌⠲⠼ ⠽⠙
7     ⠨⠅ ⠼⠢⠸⠹⠂⠌⠲⠸⠼ ⠽⠙
8   ⠰⠠⠉⠸⠲ ⠜⠷⠭⠬⠁⠘⠆⠐⠾⠬⠷⠽⠬⠁⠘⠆⠐⠾⠻
9     ⠤⠜⠷⠭⠤⠁⠘⠆⠐⠾⠬⠷⠽⠤⠁⠘⠆⠐⠾⠻
10    ⠨⠅ ⠬⠤⠆⠁ ⠸⠱
11  ⠰⠠⠙⠲ ⠠⠎⠕⠍⠍⠻⠄⠎ ⠗⠳⠞⠔⠑ ⠉ ⠆ ⠗⠑⠏⠗⠑⠎⠢⠞⠫ ⠃⠽ ⠮
12    ⠋⠕⠇⠇⠪⠬ ⠑⠭⠏⠗⠑⠎⠨⠝⠲
13      ⠠⠗⠳⠞⠔⠑ ⠰⠠⠎⠒ ⠸⠩ ⠼⠢⠬⠒⠬⠲⠷⠂⠬⠷⠤⠂⠾⠾⠬⠷⠤⠒⠾
14        ⠬⠷⠤⠢⠾⠬⠒⠷⠢⠬⠷⠤⠆⠾⠾⠬⠂ ⠸⠱
15    ⠠⠙⠗⠁⠺ ⠁ ⠎⠊⠍⠏⠇⠑ ⠙⠊⠁⠛⠗⠁⠍ ⠞⠕ ⠗⠑⠏⠗⠑⠎⠢⠞
16    ⠨⠂⠠⠗⠳⠞⠔⠑ ⠨⠆⠰⠠⠎⠲
```

Line 2: Since the anchor will not fit on one line, it begins on line 2 and is divided before the last plus sign (step b).

Line 4: A division is made before the link (step a).

Lines 6-7: This anchor will fit on one line if it starts in the runover position (cell 3). The only necessary division is made before the link on line 7 (step a).

Line 8: Since the anchor will not fit on one line, it begins on line 8 until an appropriate division site is encountered.

Line 9: Division is made before the operation sign between the radical expressions (step b).

Line 10: Division is made before the link (step a).

Line 13: Since the displayed expression will not fit on one line, it begins on line 13 until an appropriate division site is encountered.

Line 14: The runover line begins with an operation symbol that is not part of the grouped expression (step b).

PRACTICE 15C

```
1   ⠼⠁⠲ ⠠⠉⠕⠍⠏⠥⠞⠑ ⠯⠸⠌⠕⠗ ⠎⠊⠍⠏⠇⠊⠋⠽⠲ ⠸⠩
2     ⠰⠁⠸⠲ ⠠⠹⠷⠹⠒⠌⠆⠼⠾⠈⠡⠷⠹⠂⠌⠆⠼⠾⠈⠡⠷⠤⠹⠂⠌⠆⠼⠾
3       ⠠⠌⠂⠈⠡⠆⠈⠡⠒⠠⠼
4     ⠰⠃⠸⠲ ⠠⠹⠹⠙⠭⠌⠙⠞⠼⠹⠙⠘⠆⠐⠽⠌⠙⠞⠘⠆⠐⠼
5       ⠤⠹⠙⠘⠆⠐⠭⠌⠙⠞⠘⠆⠐⠼⠹⠙⠽⠌⠙⠞⠼
6       ⠠⠌⠷⠙⠭⠸⠌⠙⠞⠾⠘⠒⠐⠠⠼
7     ⠰⠉⠸⠲ ⠷⠬⠤⠾⠁⠰⠂⠊⠰⠰⠂⠐⠁⠰⠆⠊⠰⠰⠆⠐⠁⠰⠒⠊⠰⠰⠒
8       ⠐⠁⠰⠲⠊⠰⠰⠲ ⠐⠄⠄⠄ ⠁⠰⠝⠊⠰⠰⠝
9   ⠼⠆⠸⠲ ⠹⠭⠘⠆⠐⠬⠆⠌⠭⠷⠆⠭⠘⠆⠐⠬⠂⠾⠘⠆⠐⠼
10    ⠨⠅ ⠹⠠⠁⠌⠭⠼⠬⠹⠠⠃⠭⠬⠠⠉⠌⠆⠭⠘⠆⠐⠬⠂⠼
11      ⠬⠹⠠⠙⠭⠬⠠⠑⠌⠷⠆⠭⠘⠆⠐⠬⠂⠾⠘⠆⠐⠼
12    ⠨⠅ ⠹⠠⠁⠷⠲⠭⠘⠲⠐⠬⠲⠭⠘⠆⠐⠬⠂⠾⠬⠠⠃⠷⠆⠭⠘⠲⠐⠬⠭⠘⠆⠐⠾
13      ⠬⠠⠉⠷⠆⠭⠘⠒⠐⠬⠭⠾⠬⠠⠙⠭⠘⠆⠐⠬⠠⠑⠭
14      ⠠⠌⠭⠷⠆⠭⠘⠆⠐⠬⠂⠾⠘⠆⠐⠼ ⠸⠱
```

MU = Mathematical unit

Lines 2-3 [MU: fraction]: The entire complex fraction (39 cells) will not fit on one line, even if it starts on line 3 in cell 5. Division is made before the complex fraction line. The numerator fits on line 2. The denominator fits on line 3. (Review the construction of complex fractions in Lesson 8.)

Lines 4-6 [MU: fraction, numerator]: The numerator (43 cells) will not fit on one line. It is divided before the minus sign (step b within a unit). Division is also required before the fraction line (line 6). The denominator fits on one line (line 6).

Lines 7-8 [MU: base and subscript]: Division is made before a base in order to keep the unit intact. The associated baseline indicator begins the runover line (line 8). (Review the rules regarding an ellipsis on the baseline following a subscript in Lesson 6.)

Lines 9-14 [MU: anchor, links]:

Line 9 (anchor): The anchor fits on one line. It begins on the same line as the identifier. (See Section 8.22.b in Lesson 8 regarding margin rules for an itemized nested linked expression with no narrative.)

Lines 10-11 (first link) [MU: three fractions]: The first two fractions (addends) fit on line 10. Division is made before a sign of operation on the baseline (step b). Each fraction is kept intact.

Lines 12-14 (second link) [MU: fraction, numerator, grouped expressions]: The numerator must be divided. It is comprised of four addends. Addends A and B fit on line 12. A division is made before an operation sign outside of a grouping—the plus sign between addends B and C (step b within a unit). Addends C and D complete the numerator on line 13. Division is also required before the fraction line. The denominator fits on one line (line 14).

PRACTICE 15D

1 ⠼⠁⠲ ⠠⠍⠥⠇⠞⠊⠏⠇⠽ ⠘⠮ ⠋⠳⠗ ⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇⠎⠒
2   ⠸⠩ ⠷⠲⠭⠘⠔⠐⠽⠾⠷⠶⠭⠘⠦⠐⠽⠘⠆⠐⠾⠷⠢⠭⠘⠶⠐⠽⠘⠒⠐⠾
3   ⠷⠖⠭⠘⠖⠐⠽⠘⠲⠐⠾⠸⠲
4 ⠼⠆⠸⠲
5   ⠬⠤⠷⠹⠁⠘⠹⠆⠌⠒⠼⠐⠬⠁⠘⠹⠂⠌⠒⠼⠐⠃⠘⠹⠂⠌⠒⠼⠐⠬⠃⠘⠹⠆⠌⠒⠼
6   ⠐⠌⠒⠼⠾⠘⠹⠒⠌⠆⠼
7 ⠼⠒⠸⠲
8   ⠷⠂⠖⠭⠘⠲⠐⠬⠦⠭⠘⠒⠐⠽⠬⠲⠭⠘⠆⠐⠽⠘⠆⠐⠬⠆⠭⠽⠘⠒⠐⠬⠽⠘⠲⠐⠾
9   ⠷⠆⠭⠤⠽⠾
10   ⠨⠅ ⠼⠒⠆⠭⠘⠢⠐⠤⠽⠘⠢ ⠸⠱

MU = Mathematical unit

Lines 1-3 [MU: grouped expressions]: Division is made before the fourth grouping. Line 3 begins with the left parenthesis of that grouped expression.

Lines 4-6 [MU: base and exponent, grouped expression, fraction]: The fraction must be divided. The numerator will not fit on line 4 with the identifier, but it will fit on line 5. The simple fraction is divided before the fraction line. Line 6 begins with the baseline indicator associated with the fraction line. The superscript (exponent) fits on line 6. It is not separated from its base.

Lines 7-10 [MU: anchor, link, grouped expressions]: The first grouping will not fit on line 7 with the identifier, but it will fit on line 8. The anchor is divided before the second grouped expression. Although there is room on line 9 for the link, division must be made before the link (step a). Line 10 begins with the comparison symbol.

PRACTICE 15E

1 ⠼⠁⠲⠀⠸⠩⠀⠣⠒⠜⠭⠘⠂⠐⠻⠣⠖⠜⠭⠘⠆⠐⠻⠣⠂⠆⠜⠭⠘⠒⠐⠻

2 ⠀⠀⠣⠆⠲⠜⠭⠘⠲⠐⠻⠣⠲⠦⠜⠭⠘⠢⠐⠻⠀⠄⠄⠄⠀⠸⠱

3 ⠼⠃⠲⠀⠠⠮⠀⠛⠢⠻⠁⠇⠀⠎⠕⠇⠥⠰⠝⠀⠿⠀⠐⠕⠀⠗⠕⠕⠞⠀⠷⠀⠮⠀⠉⠥⠃⠊⠉

4 ⠀⠀⠑⠟⠥⠁⠰⠝⠀⠊⠎⠀⠸⠩

5 ⠀⠀⠀⠀⠭

6 ⠀⠀⠀⠀⠀⠀⠨⠅⠀⠣⠒⠜⠤⠹⠟⠌⠆⠼

7 ⠀⠀⠀⠀⠀⠀⠬⠨⠜⠹⠟⠘⠆⠐⠌⠲⠼⠬⠹⠏⠘⠒⠐⠌⠆⠶⠼⠨⠻⠻

8 ⠀⠀⠀⠀⠀⠀⠬⠣⠒⠜⠤⠹⠟⠌⠆⠼⠤⠨⠜⠹⠟⠘⠆⠐⠌⠲⠼⠬⠹⠏⠘⠒⠐⠌⠆⠶⠼⠨⠻⠻

9 ⠀⠀⠀⠀⠀⠀⠸⠱⠲

MU = Mathematical unit

Lines 1-2 [MU: radical expressions]: The first three radical expressions fit on line 1. Division is made before the fourth grouping, after the termination indicator. Line 2 begins with the index-of-radical indicator.

Lines 3-4: Item 2, text, (1-3) margins.

Lines 5-9 (displayed linked expression with margins 5-7): Working through the steps,

Step a (comparison sign on the baseline): The first division is made before the equals sign. The anchor is on <u>line 5</u>.

Step b (operation sign on the baseline): The link starts on <u>line 6</u> and is divided before the plus sign that lies between the addends. This will become <u>line 8</u>. The second addend fits all together on line 8.

Step b within a mathematical unit: The first addend does not fit on line 6. It is divided before the first plus sign. (The inner radical is kept intact.)

PRACTICE 15F

1 ⠼⠁⠲ ⠠⠐⠙⠇⠊⠣⠞ ⠲⠲⠲
2 ⠁⠲ ⠠⠮ ⠝⠥⠍⠃⠻ ⠷ ⠍⠔⠥⠞⠑⠎ ⠷ ⠐⠙⠇⠊⠣⠞ ⠿ ⠁⠝⠽
3 ⠇⠕⠉⠁⠰⠝ ⠁⠞ ⠸⠩ ⠼⠖⠴⠘⠨⠡ ⠰⠠⠝ ⠸⠱ ⠇⠁⠞⠊⠞⠥⠙⠑
4 ⠊⠎ ⠍⠕⠙⠑⠇⠫ ⠃⠽ ⠮ ⠋⠥⠝⠉⠰⠝ ⠆⠇⠂ ⠐⠱
5 ⠸⠩ ⠙ ⠨⠅ ⠙⠁⠽ ⠕⠋ ⠞⠓⠑ ⠽⠑⠁⠗⠲
6 ⠍⠷⠙⠾
7 ⠨⠅ ⠼⠒⠔⠴⠎⠊⠝ ⠈⠷⠹⠆⠨⠏⠌⠒⠖⠢⠨⠆⠢⠼
8 ⠷⠙⠤⠦⠴⠾⠈⠾
9 ⠬⠶⠒⠦ ⠸⠱

Mathematical units: link, addends, function and argument, grouped expressions, fraction

Line 5: The entire statement "d = day of the year" is mathematical. The words are uncontracted and are punctuated without the use of a punctuation indicator. (See Sections 3.2 and 3.3 in Lesson 3.)

Lines 6-9: The displayed expression begins in cell 7, with runovers in cell 9.

Line 6 (anchor): Division is made before the comparison sign (step a).

Lines 7-8 (link): The space between the function and its argument is not a preferred division site. The bracketed expression must be divided. Division is made between the factors, before the left parenthesis. This keeps the expression grouped within the inner parentheses intact.

Line 9: Division is made before the plus sign (step b) even though it would fit on line 8.

PRACTICE 15G

```
1  ⠀⠀⠠⠮ ⠎⠕⠇⠥⠰⠝⠎ ⠔⠧⠕⠇⠧⠬ ⠮ ⠙⠊⠗⠑⠉⠞ ⠋⠥⠝⠉⠰⠝⠎ ⠸⠩
2  ⠀⠀⠮⠹⠇⠕⠛⠘⠆ ⠷⠵⠬⠂⠾⠌⠵⠼⠙⠵
3      ⠨⠅ ⠇⠕⠛ ⠷⠤⠵⠾⠇⠕⠛⠘⠆ ⠷⠵⠬⠂⠾
4      ⠬⠆⠠⠇⠊⠆⠷⠵⠬⠂⠾⠇⠕⠛ ⠷⠵⠬⠂⠾⠤⠆⠠⠇⠊⠒⠷⠵⠬⠂⠾
5  ⠠⠄⠯
6    ⠮⠰⠴⠘⠠⠿
7      ⠐⠠⠹⠇⠕⠛ ⠷⠞⠬⠂⠾⠇⠕⠛ ⠷⠂⠬⠹⠂⠌⠞⠘⠆⠐⠼⠾⠠⠌⠞⠠⠼⠙⠞
8      ⠨⠅ ⠠⠉⠨⠏⠤⠹⠒⠨⠵⠷⠒⠾⠌⠦⠼ ⠸⠱
9  ⠉ ⠆ ⠋⠨⠙ ⠔ ⠠⠡⠁⠏⠞⠻ ⠼⠛⠲
```

Line 1-9: This is one paragraph interrupted twice by a displayed linked expression.
Line 1: The paragraph begins in cell 3.
Line 2: The anchor begins in cell 3. The linked expression is divided before the equals sign (step a).
Line 3: The link begins in the runover cell for displayed material (cell 5). Division is made before the plus sign that is not part of a grouped expression (step b).
Line 4: The rest of the link fits on line 4. It is not necessary to divide before the second operation sign (the minus sign).
Line 5: The paragraph continues in the runover cell for narrative (cell 1).
Line 6: The displayed expression begins in cell 3. The anchor will not fit on one line. Division is made before the opening complex fraction indicator.
Line 7: This line begins in the runover cell (cell 5) with the baseline indicator associated with the opening complex fraction indicator. The entire complex fraction is undivided. The rest of the anchor fits on this line.
Line 8: The link begins in the runover cell, cell 5 (step a).
Line 9: The paragraph concludes.

PRACTICE 15H

Option 1: The expression is divided after the termination indicator. Both fractions are on the runover line.

```
⠸⠩ ⠐⠨⠠⠎⠩⠝ ⠨⠅ ⠼⠴⠣⠠⠿⠻
⠹⠷⠁⠾⠰⠝⠐⠷⠃⠾⠰⠝⠂⠌⠷⠉⠾⠰⠝⠂⠼⠹⠵⠘⠝⠂⠌⠝⠯⠼ ⠸⠱
```

Option 2: The expression is divided before the second fraction.

```
⠸⠩ ⠐⠨⠠⠎⠩⠝ ⠨⠅ ⠼⠴⠣⠠⠿⠻⠹⠷⠁⠾⠰⠝⠐⠷⠃⠾⠰⠝⠂⠌⠷⠉⠾⠰⠝⠂⠼
⠹⠵⠘⠝⠂⠌⠝⠯⠼ ⠸⠱
```

PRACTICE 15I

Option 1: The anchor is divided after a termination indicator.

1 ⠼⠁⠲ ⠠⠮ ⠗⠑⠉⠞⠁⠝⠛⠇⠑ ⠗⠥⠇⠑ ⠿ ⠝⠥⠍⠻⠊⠉⠁⠇
2 ⠔⠞⠑⠛⠗⠁⠰⠝ ⠐⠣⠠⠮ ⠠⠍⠊⠙⠏⠕⠔⠞ ⠠⠗⠥⠇⠑⠐⠜ ⠊⠎
3 ⠔⠎⠏⠊⠗⠫ ⠃⠽ ⠮ ⠥⠎⠑ ⠷ ⠠⠗⠊⠑⠍⠁⠝⠝ ⠎⠥⠍⠎⠲ ⠸⠩
4 ⠹⠃⠤⠁⠌⠝⠼⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠴⠣⠝⠤⠂⠻
5 ⠋⠷⠁⠬⠊⠹⠃⠤⠁⠌⠝⠼⠾
6 ⠈⠱⠈⠱ ⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭ ⠸⠱

Line 4: The anchor will not fit on one line. Division is made after the termination indicator of the Sigma notation.
Line 5: The anchor is completed on this line.
Line 6: The link begins with the double tilde and fits on one line.

Option 2: The anchor is divided before the Sigma notation.

1 ⠼⠁⠲ ⠠⠮ ⠗⠑⠉⠞⠁⠝⠛⠇⠑ ⠗⠥⠇⠑ ⠿ ⠝⠥⠍⠻⠊⠉⠁⠇
2 ⠔⠞⠑⠛⠗⠁⠰⠝ ⠐⠣⠠⠮ ⠠⠍⠊⠙⠏⠕⠔⠞ ⠠⠗⠥⠇⠑⠐⠜ ⠊⠎
3 ⠔⠎⠏⠊⠗⠫ ⠃⠽ ⠮ ⠥⠎⠑ ⠷ ⠠⠗⠊⠑⠍⠁⠝⠝ ⠎⠥⠍⠎⠲ ⠸⠩
4 ⠹⠃⠤⠁⠌⠝⠼
5 ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠴⠣⠝⠤⠂⠻⠋⠷⠁⠬⠊⠹⠃⠤⠁⠌⠝⠼⠾
6 ⠈⠱⠈⠱ ⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭ ⠸⠱

Line 4: The anchor will not fit on one line. Division is made after the fraction unit.
Line 5: The Sigma notation and its argument fit all together on this line.
Line 6: The link begins with the double tilde and fits on one line.