

## **LESSON 16**

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- [INSTRUCTIONAL COMMENTARY](#)
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### **LESSON PREVIEW**

The remaining lessons take a look at a variety of formatting topics. In this lesson, spatially arranged fractions are studied, a braille format for instructional commentary is given, and layouts for stem-and-leaf plots are introduced.

**SPATIAL ARRANGEMENT OF FRACTIONS**

[NC 13.10]

Fractions are usually transcribed linearly, as discussed in Lesson 8. However, certain situations suggest or require a spatial arrangement.

Review the terminology used for the parts of a fraction.

$$\frac{3}{4}$$

*numerator*  
*fraction line*  
*denominator*

**16.1 Spatial Fraction Line**

The spatial fraction line consists of a series of dots 25. An opening fraction indicator (1456) marks the beginning of the fraction line; a closing fraction indicator (3456) signals the end.



Spatial Fraction Line with opening and closing fraction indicators (varying length)

In a spatially arranged fraction the numerator is placed above the spatial fraction line and the denominator is placed below it. The fraction line is comprised of the same number of cells as the widest item above (numerator) or below (denominator), with the fraction indicators placed one cell beyond the width on either end, as illustrated in the next section.

Note: Code switch indicators are absent in many of the isolated examples in this lesson.

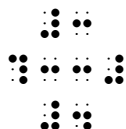
**16.2 Numerator and Denominator**

The numerator is centered above the fraction line; the denominator is centered below the fraction line. If exact centering is not possible, the item is moved to the left one cell. Since fractions do not contain material aligned for computation, the numeric indicator is used in the numerator and/or denominator according to the rules of the Nemeth code. For the same reason, the English-letter indicator is used where necessary in the numerator and/or denominator. Indicators will not necessarily align; centering is the objective.

**Example 16-1**

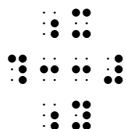
(a)

$$\frac{3}{4}$$



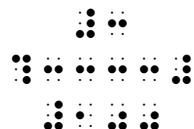
(b)

$$\frac{x}{y}$$



(c)

$$\frac{3}{100}$$



(d)

$$\frac{25}{100}$$

(e)

$$\frac{5x}{3}$$

### 16.3 Placement of Identifiers with Spatially Arranged Fractions

An identifier, if present, is placed on the same braille line as the fraction line. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement.

*Example 16-2*

5.  $\frac{(9x) \div 3}{(3x+15) \div 3}$

6.  $\frac{3xy+9x^2}{3x}$

*Three blank cells come between the rightmost symbol of an arrangement and the next identifier. (Lesson 9)*

## *Situations Requiring Spatial Presentation of Simple Fractions*

### 16.4 Fractions Arranged Spatially for Illustration

When the parts of a simple fraction are labeled, use a spatial arrangement. The labels are uncontracted because they are within the code switches.

*Example 16-3*

$\frac{3}{4}$	<i>numerator</i> <i>fraction line</i> <i>denominator</i>	
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*Example 16-4*

Four children share a dozen cookies equally. Expressed as a fraction, how many cookies does each child receive?

$$\frac{12}{4} \quad \begin{array}{l} \text{number of cookies} \\ \text{number of children} \end{array}$$

1	
2	
3	
4	
5	
6	
7	
8	
9	

### PRACTICE 16A

1. Shandra invited seven friends to a pizza party. Two pizzas were ordered. Each pizza had eight slices. Which fraction shows how many slices of pizza each child can have if they share equally?
- $\frac{2}{8}$       number of pizzas  
              number of children
  - $\frac{8}{8}$       number of slices  
              number of children
  - $\frac{16}{8}$      number of slices  
              number of children

### 16.5 Cancellation with Replacement Values [NC 12.1]

A spatial arrangement is required when numbers, letters, or abbreviations are canceled in print and replacement values are shown. *Reminder:* The print copy may show any type of stroke to indicate cancellation.

⋮	Opening Cancellation Indicator
⋮⋮	Closing Cancellation Indicator

When cancellation occurs within a fraction, replacement items are placed above canceled items in the numerator and below canceled items in the denominator. Each replacement item is centered with respect to the canceled term. If exact centering is not possible, the term is moved to the left one cell.

#### Example 16-5

	1	⋮ ⋮
	2	⋮ ⋮ ⋮ ⋮
$\frac{1}{\cancel{25}}$	3	⋮ ⋮ ⋮ ⋮ ⋮ ⋮
$\frac{50}{2}$	4	⋮ ⋮ ⋮ ⋮
	5	⋮ ⋮

Line 1: Replacement "1" is centered above the canceled number. A numeric indicator is required.

Line 2: A numeric indicator is not required because the numeral is not preceded by a space.

Line 3: The fraction line is as long as the widest item in the fraction. In this case, both numerator and denominator have a 4-cell item.

Line 4: A numeric indicator is not required because the numeral is not preceded by a space.

Line 5: Replacement "2" is centered below the canceled number. A numeric indicator is required.

### Example 16-6

$$\frac{\overset{1}{\cancel{25}}}{\underset{2}{\cancel{50}}} = \frac{1}{2}$$

Note that the second fraction ("one-half") is transcribed as a linear fraction because it doesn't contain cancellation. The equals sign and the linear fraction are placed on the same line as the spatial fraction line.

- 16.5.1 **Alignment Considerations with Cancellation.** Vertical alignment of digits or of indicators is not a consideration in spatially arranged fractions when the material is not aligned for computation. Replacement items are centered above or below their canceled item. Placement of the replacement item is relative to the width of the canceled item, without respect to place value. Note the location of the replacement items in [Example 16-7](#).

### Example 16-7

$$\frac{\overset{7}{\cancel{1449}}}{\underset{1}{\cancel{207}}} = 7$$

Replacement digit 7 is exactly centered above the canceled number 1449. The denominator is "left-centered" beneath the fraction line. Replacement digit 1 is left-centered beneath the canceled number 207.

- a. **Placement of the Numeric Indicator.** When the canceled number and its replacement contain the same number of digits, the numeric indicator is placed above (or below) the opening cancellation indicator.

Example 16-8

$$\frac{\overset{1}{\cancel{6}}}{\underset{6}{\cancel{36}}} = \frac{1}{6}$$

*Replacement digit 1 is left-centered above the canceled number 62. The numeric indicator lies directly above the opening cancellation indicator. Canceled number 6 (the numerator) is left-centered above the fraction line. Canceled number 36 (the denominator) is centered below the fraction line. Replacement digit 6 is centered below the canceled number above it.*

- 16.5.2 **Stacked Cancellation.** More than one level of cancellation may be encountered.

Example 16-9

$$1. \frac{\overset{1}{\cancel{15}} \times \overset{22}{\cancel{5280}}}{\underset{\cancel{4}}{\cancel{60}} \times \underset{1}{\cancel{60}}}$$

*The numerator and denominator are each centered above/below the fraction line. Each replacement number is centered (or left-centered) above or below its canceled number.*

## PRACTICE 16B

Simplify using cross canceling. Then multiply the numerators and multiply the denominators.

$$1. \frac{\overset{1}{\cancel{3}}}{\cancel{15}_3} \times \frac{\overset{1}{\cancel{5}}}{\cancel{6}_2} = \frac{1}{6}$$

$$2. \overset{1}{\cancel{2}} \times \frac{8}{\cancel{2}_1} = \frac{1 \times 8}{1} = 8$$

16.5.3 **Cancellation and Level Indicators.** Care must be taken to place the opening cancellation indicator and its paired closing cancellation indicator on the same level.

$$\gg \cancel{b^x} \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$$

*Both the base and its superscript are canceled with one stroke. A baseline indicator places the cancellation terminator on the same level as the paired opening cancellation indicator.*

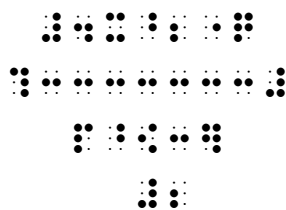
$$\gg b^{\cancel{x}} \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$$

*Only the superscript is canceled. The opening and closing cancellation indicators are at the superscript level.*

Example 16-10

$$\frac{\overset{a}{\cancel{a}}}{\cancel{a}_1} = a \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$$

Example 16-11

$$\frac{4x^2q}{p^{\cancel{2}^2}}$$


*Replacement digit 2 is placed below the canceled superscript in the denominator, even though it is printed beside the canceled superscript.*

**PRACTICE 16C**

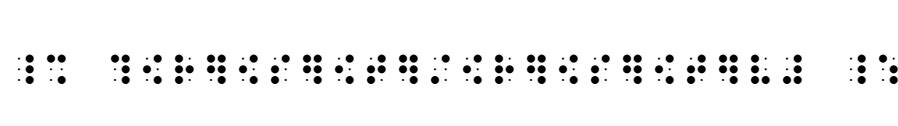
$$1. \quad \frac{2b+a}{b} \div \frac{4b^2-a^2}{b^2} = \frac{\overset{1}{\cancel{(2b+a)}}}{\underset{1}{\cancel{b}}} \cdot \frac{\overset{b}{\cancel{b^2}}}{\underset{1}{\cancel{(2b+a)}}(2b-a)} = \frac{b}{2b-a}$$

**16.6 Cancellation Without Replacement Values [NC 12.1]**

Material containing cancellation with no replacement may be transcribed linearly.

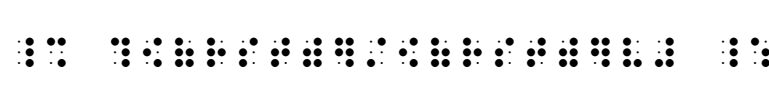
16.6.1 **Linear Presentation of Simple Fractions with Canceled Items.** When replacement items are not given for canceled material in the numerator and denominator, a linear fraction may be transcribed. The cancellation indicators must represent exactly what is canceled in print.

Example 16-12

$$\frac{\cancel{r} \cancel{s} \cancel{t}}{\cancel{r} \cancel{s} \cancel{t} v}$$


*Each letter "r" "s" and "t" is printed with a slash through it.*

Example 16-13

$$\frac{\cancel{(rst)}}{\cancel{(rst)}v}$$


*Each letter combination "rst" as well as the parentheses are printed with a line through them.*







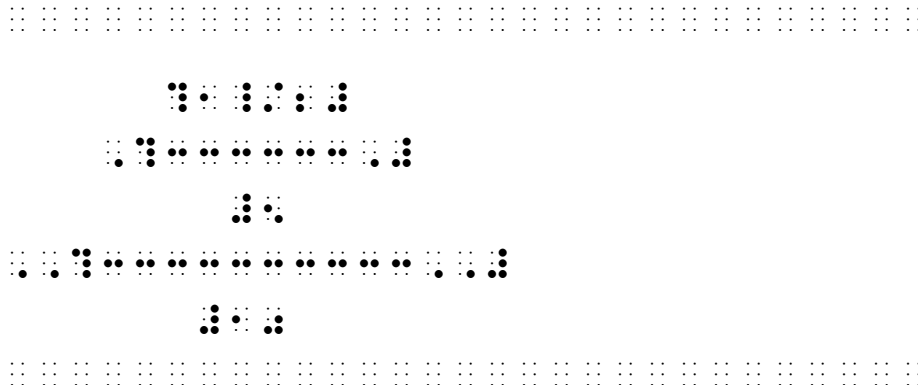




Example 16-18

$$\frac{\frac{1}{2}}{\frac{5}{10}}$$

(entirely spatial)



(entirely linear)

**16.9 Higher Orders of Complexity**

Hypercomplex fractions of higher order may be transcribed in the manner described in [Section 16.8](#) with dot 6 added the appropriate number of times before the fraction indicators and the matching fraction lines. No examples are shown.

**PRACTICE 16E**

*Instructions:* Transcribe this hypercomplex fraction using the combined method—that is, transcribe each complex fraction as a linear arrangement within the overall spatial arrangement. Numerators and denominators can be determined by noting the length of each fraction line in print.

1. Solve for  $r$  in terms of  $s$ .

$$\frac{\frac{\frac{r^2 - 4s^2}{s^2}}{r + 2s}}{\frac{s}{4r - 2s^2}} \cdot \frac{3s}{\frac{2s^2 - 3r}{4r}}$$

**CONTINUED FRACTIONS**

[NC 13.9]

**16.10 Definition and Recognition**

A continued fraction is a fraction within a fraction. Each denominator is the sum of a whole number and a fraction. The digit in the denominator of one fraction becomes the digit in the numerator of the next fraction. You may find them in the study of irrational numbers such as pi and certain square roots.

The continued fraction shown below can be described as follows. The fraction begins with  $1 - \frac{7}{3}$ . The first denominator is  $3 + \frac{3}{2}$ . The next denominator is  $2 + \frac{2}{2}$ . The final denominator is  $2 - \frac{2}{3}$ .

$$n = 1 - \frac{7}{3 + \frac{3}{2 + \frac{2}{2 - \frac{2}{3}}}}$$

**16.10.1 Transcription Rules.** A continued fraction is transcribed as a spatial arrangement. The layout mimics the print arrangement. Each fraction line begins in the cell after the plus or minus sign (unspaced). Opening and closing fraction indicators are not used. All fraction lines end in the same cell, aligned with the rightmost symbol in the arrangement.

Each new denominator's whole number is aligned directly below the first cell of the separation line above it. Its numerator is centered with respect to the fraction line below it. If that number cannot be exactly centered, it is placed one cell to the left of center.

The rules will make sense when you study [Example 16-19](#) which is the transcription of the continued fraction shown in [Section 16.10](#), above.



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**PRACTICE 16F**

The square root of 3 can be expressed as a continued fraction.

$$\sqrt{3} = 1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}$$

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**INSTRUCTIONAL COMMENTARY****[NC 4.8.9 and 26.4.5]****16.11 Format for Instructional Commentary**

When instructional commentary appears alternated with math problems, place the comment on the line following the related expression, blocked 4 cells to the right of the runover position for the expression. Explain this format in a transcriber's note. Sample transcriber's note:

Comments printed beside related math problems are placed on the line following the expression, blocked four cells to the right of the runover position of the expression.

When switching into or out of Nemeth before a change of margins, place the switch indicators after the last item of the line rather than at the beginning of the next line. This provides clarity within the indented margin pattern. (See [Example 16-21](#))

The single-word switch indicator is used, as needed.

Comments are often printed in a distinct color or typeface. (In the examples in this section, instructional commentary is printed in italics.) The variant typeform is disregarded in the braille transcription.

Notice in the example transcriptions how the comments are clearly set off by indentation.

Example 16-20

If  $y$  varies inversely as  $x$ , and  $y = 3$  when  $x = 4$ , find  $y$  when  $x = 18$ .

$$\frac{x_1}{y_2} = \frac{x_2}{y_1} \quad \textit{Given.}$$

$$\frac{4}{y_2} = \frac{18}{3} \quad \textit{Substitute the known values.}$$

$$18y_2 = 12 \quad \textit{Now cross multiply.}$$

$$y_2 = \frac{12}{18} \textit{ or } \frac{2}{3} \quad \textit{Divide each side by 18 and simplify.}$$

The value of  $y$  when  $x = 18$  is  $\frac{2}{3}$ .







Lines 6, 11, 14: Each comment begins in cell 13, which is four cells to the right of the runover position of the previous line, even if no runover is present.

Lines 7-8, 12, 15: Runovers to comments are blocked (cell 13, in this case).

Lines 9-10: Because the canceled numbers do not have replacements, the fraction is transcribed linearly.

### PRACTICE 16G

*Instructions:* Transcribe this as one paragraph with four lines of displayed material between the two sentences.

What percent of 48 is 54?

$$n \cdot 48 = 54$$

*Write an equation.*

$$\frac{48n}{48} = \frac{54}{48}$$

*Divide each side by 48.*

$$n = 1.125$$

*Simplify.*

$$= 112.5\%$$

*Change the decimal to a percent.*

*Answer:* 54 is 112.5% of 48.

## **STEM-AND-LEAF PLOTS**

### **16.12 Recognition**

A stem-and-leaf plot is a method of showing data distribution in columns and rows. A vertical line partitions the information into data on the left, called the stem, and data on the right, called the leaf. Stem and leaf data may consist of numbers, letters, and/or blank entries and may be spaced or unspaced in the print copy. Stem data are right justified to the vertical line; leaf data are left justified to the vertical line. The columns may include a heading. Here is a sample.

Stem	Leaf
10	0 4
9	3 4 5 7 8 9
8	0 2 2 9
7	5 8 9
6	3

*A distinctive feature of a stem-and-leaf plot is the vertical line separating the stems from the leaves.*

### **16.13 The Table**

A stem-and-leaf plot is transcribed in Nemeth notation using the guidelines for tables outlined in *Braille Formats*. The entire table is transcribed in Nemeth, including the column headings (if present), even if the numbers or letters are unmodified. Column headings begin in cell 1 and no contractions are used. Column separation lines follow on the next line below the headings, covering the same number of cells as the widest data entry in each column. Column headings are not transcribed if they do not appear in print.

The vertical line between the stem column and the leaf column is transcribed as dots 456, preceded and followed by one blank cell. This symbol is transcribed in every row, including between column headings and between column separation lines.

⠠⠨⠠ Vertical Line (preceded and followed by a space)

The following Nemeth rules apply to [Example 16-23](#).

- The numeric indicator is omitted in the body of the plot.
- Spaces are not inserted between single-digit entries.
- The data are justified to the vertical line as shown in print.









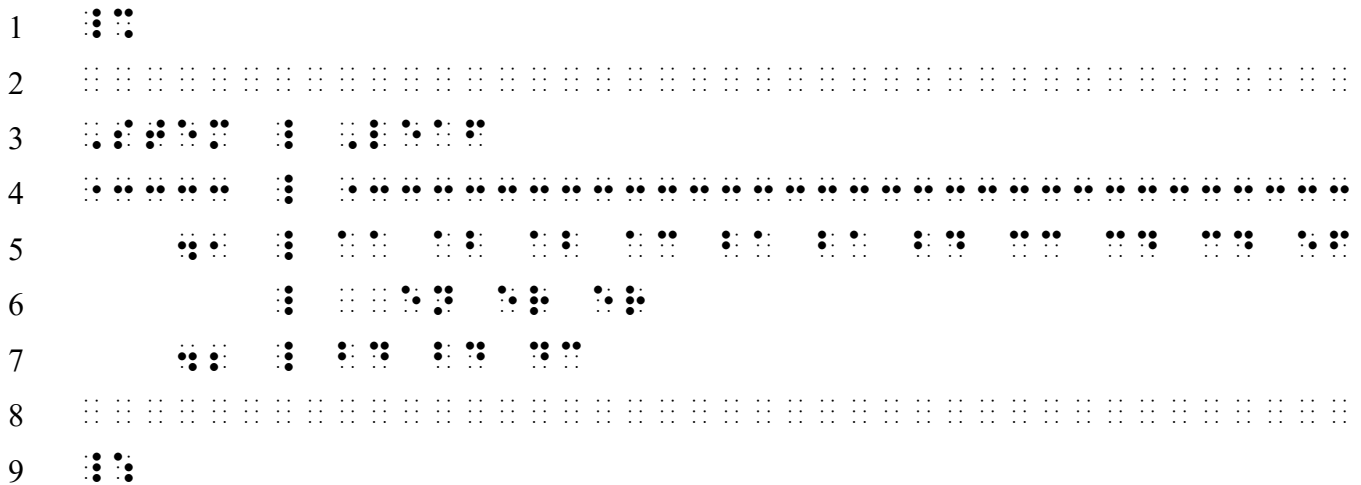




16.18.2 **Braille Runovers.** If leaf data will not fit on one line in the braille transcription, indent the runover line two cells to the right after using as much of the braille line as possible. Do not repeat the stem row heading. [Example 16-30](#) illustrates, expanding the data shown in row one of [Example 16-27](#).

Example 16-30

Stem	Leaf
41	aa ab ab ac ba ba bd cc cd cd ef en er er
42	bd bd dc



Lines 3-7: The vertical line is transcribed on every line in the table.

**16.19 Back-To-Back Plot**

A back-to-back stem-and-leaf plot is used when two sets of data are being compared. There are three columns. The stem is the middle column. Data is read from the stem outward, which means that data in the left leaf is read from right to left. There will be two keys. Here are the rules as they apply to [Example 16-31](#).

- *The Keys:* The left column's key is transcribed first, followed on the next line by the right column's key.
- *The Columns:* Column separation lines are transcribed below only the lines that contain column headings.
- *The Runovers:* Right leaf data are read from left to right; runovers to the right leaf data column are shifted two cells to the right. Left leaf data are read from right to left; runovers to the left leaf data column are shifted two cells to the left.

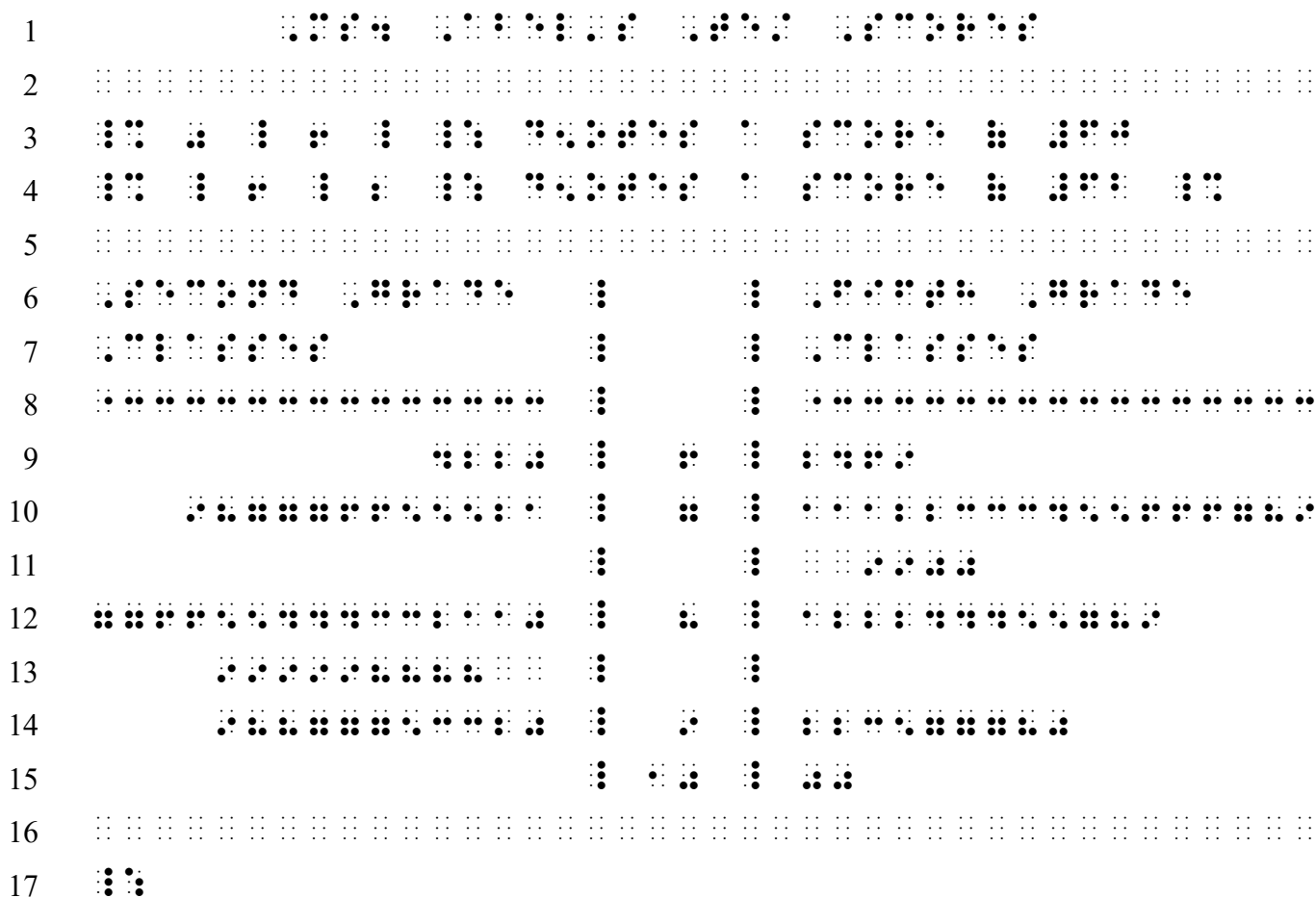
Example 16-31

**Ms. Abel's Test Scores**

Second Grade Classes		Fifth Grade Classes
4220	6	2469
987776655521	7	111223334556667899900
999998888776655444332110	8	122244455789
98877753320	9	223577780
	10	00

0 | 6 | denotes a score of 60

| 6 | 2 denotes a score of 62



Line 3: Key to left column. *Italic typeform is disregarded.*

Line 4: Key to right column. *Italic typeform is disregarded.*

Line 5: A blank line precedes the plot.

Lines 6-7: Column headings are uncontracted in a stem-and-leaf plot. These column headings require two lines. Runovers to column headings are blocked.

Line 8: The stem column (middle column) has no heading, so no column separation line is transcribed.

Line 11: Reading left to right, runovers to the right leaf data column are shifted two cells to the right.

*Line 13: Reading right to left, runovers to the left leaf data column are shifted two cells to the left.*

*Line 15: The blank left-leaf entry in the last row is a blank space in braille.*

*Line 16: The blank line is required following the table.*

*Line 17: The Nemeth Code terminator is alone on this line, outside of the table arrangement.*

### PRACTICE 16H

A pet store owner constructed the following stem-and-leaf plot showing the number of guinea pigs at each of her seventeen stores.

Stem	Leaf
0	7 8
1	
2	0 6 8 8 8
3	0 2 6 6 7 8
4	1 2 6 6
5	
<i>Key: 2   0 represents 20 guinea pigs</i>	

How many stores have fewer than 36 guinea pigs?

*For further practice, see Addendum 1—Reading Practice.*

Submit Exercise 16 to your instructor.

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