LESSON 5

SIGNS OF OPERTION

SIGNS OF COMPARISON

Format

Nemeth Instructions

Simple Tables

Answers to Practice Material

LESSON PREVIEW

Many more operation signs and signs of comparison are explored, including negated forms. Table format is introduced, with a table consisting of mathematical symbols and their names. Mathematical use of the colon meaning "such that" is shown. The concept of symbols compounded vertically and of symbols compounded horizontally is introduced with certain signs of comparison. Considerations for format of instructions are investigated.

This lesson introduces many more symbols that you will come across in all areas of mathematics. Look carefully at the print symbol as many look similar to each other. Understand the context – some symbols are used as signs of operation as well as signs of comparison. The function of the symbol must be determined in order to apply proper spacing rules.

SIGNS OF OPERATION, cont.

5.1 Review of Signs of Operation

In Nemeth, no space is left before or after a sign of operation unless it is preceded or followed by a sign of comparison, an ellipsis, a dash, an unrelated word, or an abbreviation with a related value. Signs of operation are mathematical symbols and are punctuated accordingly. The following signs of operation have already been introduced.

⠬	Plus	$+$
⠤	Minus	$-$
⠈⠡	Multiplication Cross	$\times$
⠡	Multiplication Dot	$\cdot$
⠨⠌	Division	$\div$

5.2 Signs of Operation Using Plus and Minus

Plus and minus signs printed one atop the other are considered to be one symbol. In braille, they are written side by side, with the top symbol transcribed first. This two-cell symbol must never be divided between braille lines.

⠬⠤	"Plus or Minus"	$\pm$
⠤⠬	"Minus or Plus"	$\mp$

⫸ 38 ± 7 ⠼⠒⠦⠬⠤⠶

⫸ 38 ∓ 7 ⠼⠒⠦⠤⠬⠶

Example 5-1

± 5 means $+5$ or -5.

⠸⠩ ⠬⠤⠢ ⠠⠄⠍⠂⠝⠎ ⠬⠢ ⠠⠄⠕⠗ ⠤⠼⠢ ⠸⠱⠲

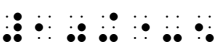
Review the rules in Section P8 of the Preliminary Lesson regarding the use/nonuse of the numeric indicator with positive and negative numbers.

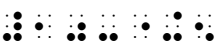
Example 5-2

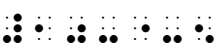
Can $3 \pm 1 = +4$ *and* $+2$?

When plus and minus signs are printed side by side, the multipurpose indicator (dot 5) prevents these particular symbols from being read as "plus or minus", "minus or plus", or a dash.

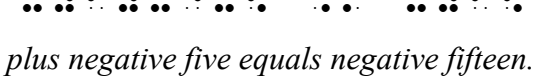
⠸	Multipurpose Indicator	
⠸⠬⠐	Plus followed by Minus	$+-$
⠸⠤⠐	Minus followed by Plus	$-+$
⠸⠤⠐	Minus followed by Minus	$--$

⋙ $10 + -5$ 

⋙ $10 - +5$ 

⋙ $10 - -5$ 

The multipurpose indicator is not used between other side-by-side operation symbol constructions because they will not be misread. Compare:

⋙ $-10 + -5 = -15$ 

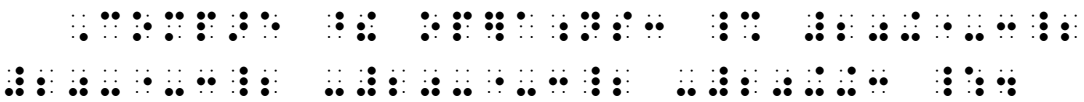
Spoken: Negative ten plus negative five equals negative fifteen.

⋙ $-10 + +5 = -5$ 

Spoken: Negative ten plus positive five equals negative five.

Example 5-3

Compare these operations: $20 + -3$; $20 - -3$; $-20 - -3$; $-20 + +3$.



5.3 Signs of Operation That Look Like Literary Characters

Some mathematical signs use characters also seen in literary writing. When used in literary context with no mathematical meaning, the symbols and rules of UEB apply. When used in mathematical context, the Nemeth symbols shown in the box are used. As mathematical operation signs, the spacing rules for operation symbols are followed.

⠸⠯	Ampersand	&
⠈⠼	Asterisk	*
⠨⠼	Crosshatch	#
⠸⠻	Dagger, Single	†
⠸⠸⠻	Dagger, Double	‡
⠈⠠⠏	Paragraph Mark	¶
⠈⠠⠎	Section Mark	§

5.3.1 **Ampersand**

⠸⠯

The ampersand functioning as a symbol of operation is commonly encountered in the study of logic, where it means "and". The symbol may be referred to as "logical conjunction".

Example 5-4

One can define F as p & $-$ p for any proposition p where "&" is *logical conjunction* and $-p$ is "not p".

⠀⠀⠠⠐⠕ ⠉ ⠙⠑⠋⠔⠑ ⠰⠠⠋ ⠵ ⠸⠩ ⠏ ⠸⠯ ⠤⠏ ⠸⠱ ⠿ ⠁⠝⠽
⠏⠗⠕⠏⠕⠎⠊⠰⠝ ⠰⠏ ⠐⠱ ⠸⠩ ⠦⠸⠯⠸⠴ ⠸⠱ ⠊⠎ ⠨⠂⠇⠕⠛⠊⠉⠁⠇
⠨⠂⠒⠚⠥⠝⠉⠰⠝ ⠯ ⠸⠩ ⠤⠏ ⠸⠱ ⠊⠎ ⠦⠝ ⠰⠏⠴⠲

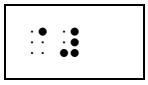
Example 5-5

Create a table to compare the price of bananas at the A & P with those at Price Chopper.

⠀⠀⠠⠉⠗⠂⠞⠑ ⠁ ⠞⠁⠃⠇⠑ ⠞⠕ ⠉⠕⠍⠏⠜⠑ ⠮ ⠏⠗⠊⠉⠑ ⠷
⠃⠁⠝⠁⠝⠁⠎ ⠁⠞ ⠮ ⠠⠁ ⠈⠯ ⠰⠠⠏ ⠾ ⠘⠹ ⠁⠞ ⠠⠏⠗⠊⠉⠑
⠠⠡⠕⠏⠏⠻⠲

This ampersand is used as a literary device, using the UEB symbol and following the spacing rules of UEB.

5.3.2 **Asterisk**



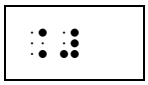
a. The asterisk as an operation symbol is commonly encountered in material about calculators where the symbol represents multiplication. When a numeral follows an asterisk, the numeric indicator is used even though the numeral is not preceded by a space.

⋙ $3 * 9 = 27$ 

⋙ $.3 * .9 = 0.27$ 

b. The asterisk used as a reference marker will be discussed in Lesson 13.

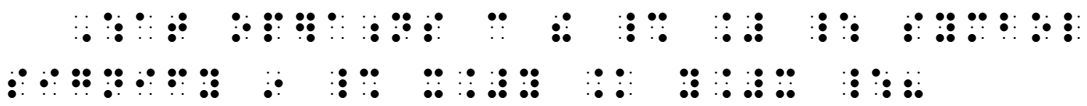
5.3.3 **Crosshatch**



a. When the crosshatch represents a symbol of operation, the Nemeth symbol is used.

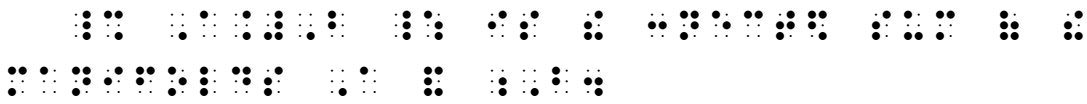
Example 5-6

What operations can the # symbol signify in $x\#y = y\#x$?

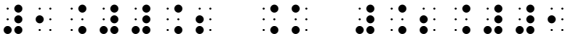


Example 5-7

A#B is the connected sum of the manifolds A and B.



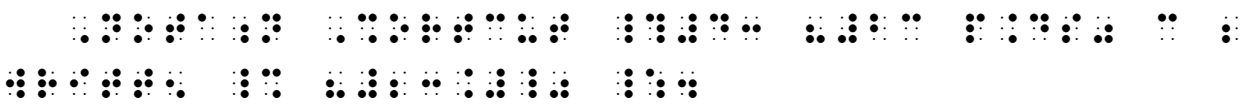
b. When a numeral or a decimal point and a numeral follows a crosshatch, the numeric indicator is transcribed even though the numeral is not preceded by a space.

⋙ $1 \# .2 = .2 \# 1$ 

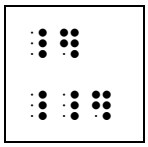
c. When the crosshatch is used in literary context such as "Problem #1" or as a hashtag in a social media reference or URL, the UEB symbol is used. When the crosshatch denotes pounds (weight), the Nemeth symbol is used.

Example 5-8

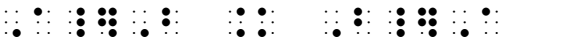
Notation Shortcut #4: "23 pounds" can be written "23#".



5.3.4 Dagger and Double Dagger



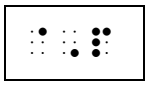
a. The dagger and double dagger may be used as operation symbols in binary operations.

⋙ A † B = B † A 

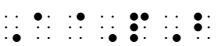
⋙ A ‡ B = B ‡ A 

b. The dagger used as a reference marker will be discussed in Lesson 13.

5.3.5 Paragraph Mark

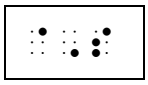


In mathematical context, the Nemeth symbol is used. A numeric indicator is required before a numeral following a paragraph mark.

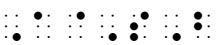
⋙ A ¶ B 

⋙ 3 ¶ 4 = 4 ¶ 3 

5.3.6 Section Mark



In mathematical context, the Nemeth symbol is used. A numeric indicator is required before a numeral following a section mark.

⋙ A § B 

⋙ 3 § 4 = 4 § 3 

PRACTICE 5A

Instructions: Before transcribing the practice, review the spacing rules for operation signs in Section 5.1 and in Section P8.1 in the Preliminary Lesson. Transcribe this entire list in Nemeth.

$4 \pm 1,\ 400 \pm 10,\ 6 \mp 1,\ 600 \mp 10$

$\mu \pm 1.645\ \sigma$

$50 - +5 = 45$

$50 + -5 = ?$

$-3 - -3 = 0$

$\mathrm{A} \,\&\, \mathrm{B} = \mathrm{B} \,\&\, \mathrm{A}$

$a * (b * c) = (a * b) * c$

$(1 + 2) * (3 + 4) = 3 * 7$

$\#\mathrm{A} = \#\mathrm{B}$

$.5\#.9 = .9\#.5$

$[(p \dagger p) \dagger (q \dagger q)]$

$s \P t = u \P v$

$1 \P 3 = 4 \P 3$

$m \S y = y \S m = y$

$5 \S 6 = 6 \S 5 = 6$

5.4 Signs of Operation Unique to Mathematics

⠸⠡	Backslash	\
⠡	Dot	·
⠨⠡	Hollow Dot	∘
⠨⠩	Intersection ("cap")	∩
⠈⠩	Logical Product	∧
⠈⠬	Logical Sum	∨
⠨⠤	Minus with Dot Over	∸
⠸⠌	Slash	/
⠈⠱	Tilde, Simple	~
⠈⠠⠱	Tilde, Extended	∼∼
⠨⠬	Union ("cup")	∪
⠳	Vertical Bar	|
⠌⠳	Vertical Bar, Negated	∤ or ∤

Examples of each symbol are shown in Sections 5.4.1-5.4.12. Note that, as with the other operation signs you have learned, these signs are unspaced from related mathematical terms regardless of the spacing shown in print.

5.4.1 Backslash

⠸⠡

The backslash slants upper left to lower right. In the context of operation signs, the backslash means "divides" or "is a factor of".

Example 5-9

$b\backslash a$ can be read as "b divides a."

⠸⠩ ⠃⠸⠡⠁ ⠸⠱ ⠉ ⠆ ⠗⠂⠙ ⠁⠎ ⠦⠰⠃ ⠙⠊⠧⠊⠙⠑⠎ ⠁⠲⠴

Example 5-10

3\6 denotes "3 is a factor of 6."

⠸⠩ ⠼⠒⠸⠡⠖ ⠸⠱ ⠙⠢⠕⠞⠑⠎ ⠦⠼⠉ ⠊⠎ ⠁ ⠋⠁⠉⠞⠕⠗ ⠷
⠼⠋⠲⠴

5.4.2 **Dot**

⠐

In addition to operating as a multiplication sign, the dot may also be used to denote "and" in the study of logic. In either case, the symbol is transcribed without a space.

Example 5-11

In logic, $p \cdot q$ is read "p and q".

Example 5-12

Torque is expressed as $\mathrm{N} \cdot \mathrm{m}$.

An English-letter indicator is required for the single-letter abbreviations N (Newton) and m (meter). See Section 4.21 in Lesson 4. The operation sign is not spaced because N and m have no related value. See Section 4.20.1.b in Lesson 4.

5.4.3 **Hollow Dot**

⠡

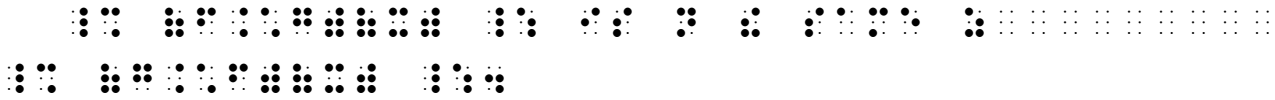
The hollow dot may be used as a sign of operation. It is also seen in function notation. The raised hollow dot used to represent degrees will be discussed in Lesson 6.

Example 5-13

$a \circ (b \circ c) = (a \circ b) \circ c$

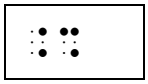
Example 5-14

$(f \circ g)(x)$ is not the same as $(g \circ f)(x)$.



Be sure to transcribe the hollow dot symbol – this is not the letter "o".

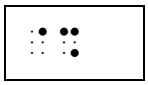
5.4.4 **Intersection**



This operation symbol is also called a "cap".

⋙ $A \cap B = B \cap A$

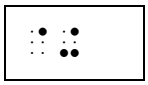
5.4.5 **Logical Product**



In the study of logic, this operation sign means "and" or "meet".

⋙ $p \wedge q \wedge r$

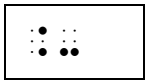
5.4.6 **Logical Sum**



In the study of logic, this operation sign means "or" or "join".

⋙ $p \vee q \vee r$

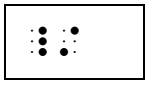
5.4.7 **Minus with Dot Over**



This operation sign means "proper difference".

⋙ $x \dot{-} y = 0$

5.4.8 **Slash**

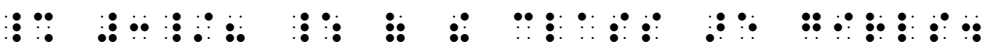


The term "slash" refers to the forward slash, which slants from lower left to upper right. In Nemeth, no space is left between the slash and any numeral, word, part of a word, or abbreviation to which it applies.

a. With Numerals When a slash represents a fraction line in a fraction where the numerator and denominator are printed on the baseline of writing, a switch to Nemeth is required. The numeric indicator is not needed for a numeral immediately following a slash.

Example 5-15

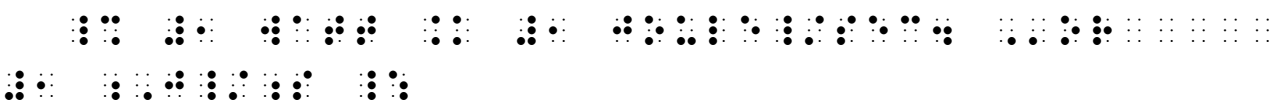
3/8 of the class are girls.



b. With Words When a slash between words or abbreviations means "divided by", "per", or "over", the slash is a mathematical operation sign and a switch to Nemeth is required.

Example 5-16

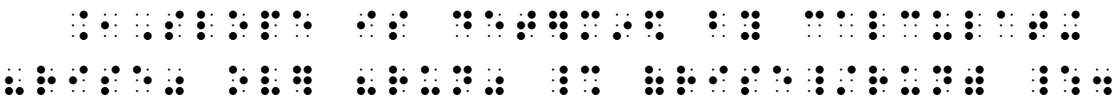
1 watt = 1 joule/sec. or 1 J/s



This slash means "per" (joule per second). A single-letter abbreviation in Nemeth requires an English-letter indicator, even when touching a slash. See Section 4.21 in Lesson 4.

Example 5-17

Slope is determined by calculating "rise" over "run" (rise/run).

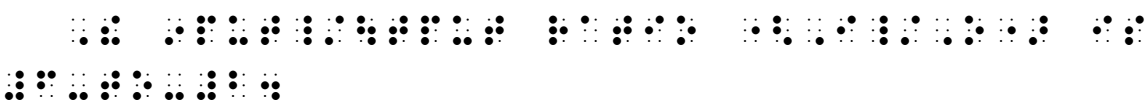


This slash means "over".

c. Slash in Literary Context UEB rules for the solidus (forward slash) are followed when the slash does not mean "divided by", "per", or "over".

Example 5-18

The input/output ratio (I/O) is 6-to-2.



This slash means "to" and so is transcribed in UEB.

Example 5-19

The chapter test will be given on Friday, 10/23. Bring your calculator and/or iPad to class. Practice problems can be found at https://www.Math.edu/Chapter12/Practice.html.

All of these slashes are in literary context.

5.4.9 Tilde and Extended Tilde

a. The simple tilde has one peak. In logic, the tilde may be used as a sign of operation meaning "not".

> ⫸ $\sim p$

> ⫸ $\sim(\sim p)$

b. When two symbols for the tilde follow one another, a multipurpose indicator (dot 5) is inserted between them to indicate that they are written horizontally.

> ⫸ $\sim\sim\ p \vee q$

c. The extended tilde has more than one peak.

> ⫸ $\backsim\!\backsim\ s \vee t$

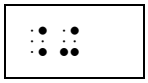
d. When the tilde symbol is used to replace the word "approximately", "about", or "around" it is transcribed as an unspaced symbol.

Example 5-20

The test will begin in ~15 seconds.

e. The tilde is also used as a sign of comparison. Consider the context to determine its meaning. (See Section 5.6.12.) When the tilde's meaning cannot be determined from context, follow print spacing.

5.4.10 Union



This operation symbol is also called a "cup".

⋙ $A \cup B = B \cup A$ 

5.4.11 Vertical Bar

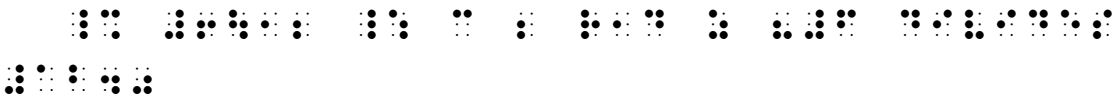
a. When the vertical bar means "is a factor" or "divides", it is functioning as an operation sign.

Example 5-21

In $b|a$, b is a factor of a.

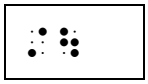
Example 5-22

6|12 can be read as "6 divides 12."

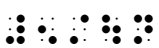


b. The vertical bar is also used as a sign of grouping (Lesson 2) and as a sign of comparison (Section 5.6.14). It is also used in probability notation (Lesson 13). Consider the context to determine its meaning. When the vertical bar's meaning cannot be determined from context, follow print spacing.

5.4.12 Vertical Bar, Negated



This symbol means "does not divide".

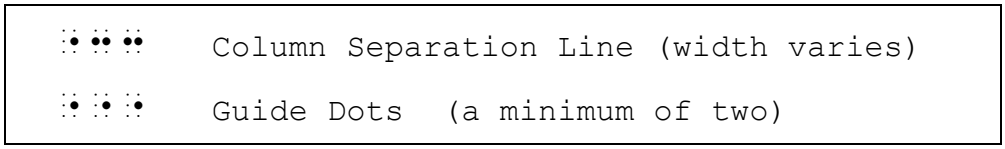
⋙ $5 \nmid n$ 

Format: Simple Tables

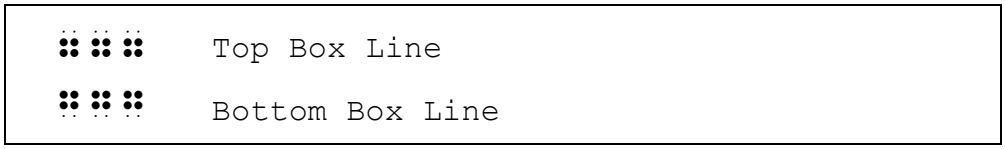
5.5 Introduction to Table Format

In this lesson, we look at a simple table where the items in one column must be transcribed in Nemeth. General guidelines for the layout of tables are given in *Braille Formats*. There you will find the definition of a table, margins used, column separation lines, space between columns, use of guide dots, and considerations when a table is too wide to fit on the braille page.

The following symbols may be used in a table, in either UEB or Nemeth.



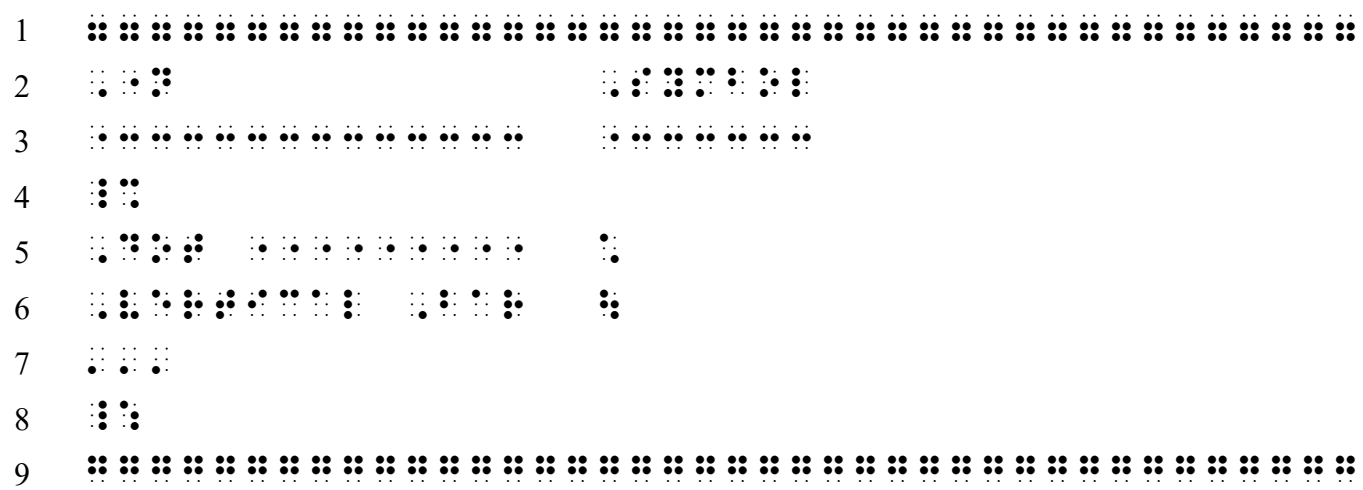
Box lines may also be used in either code.



When a substantial number of table entries consist of technical material, the recommendation is that the entire body of the table be transcribed in Nemeth, including any words. One opening Nemeth Code indicator precedes row 1 and one Nemeth Code terminator is placed at the end of the table, in cell 1. Words within the body of the table are transcribed without contractions, and the single-word switch indicator is not used. Further details regarding tables will be covered in Lesson 18.

Here is the layout you will use in PRACTICE 5B, which begins like this:

Name	Symbol
Dot	•
Vertical Bar	|



Line 2: The column headings are not mathematical. Contractions are used.
Line 3: Column separation lines are inserted according to Braille Formats guidelines.
Line 4: The opening Nemeth Code indicator is placed in cell 1.
Line 5: The first row is transcribed in Nemeth. Guide dots are inserted according to Braille Formats guidelines.
Line 6: Words in Nemeth are uncontracted.
Line 7: The ellipsis indicates that there will be further entries in your transcription.
Line 8: Terminate Nemeth on the line after the completion of the table, in cell 1.

5.5.1 **Omissions in a Simple Table.** In a table transcribed in Nemeth, when a dash, underscore, ellipsis, or other omission symbol is printed in an otherwise blank entry field, the appropriate Nemeth symbol is transcribed. (See Lesson 1.) Guide dots are inserted, as needed. When the entry field is blank, follow Braille Formats regarding the insertion of a series of guide dots across the width of a column.

PRACTICE 5B

Instructions: Include the box lines for this table.

Name	Symbol
Dot	•
Vertical Bar	|
Logical Product	∧
Simple Tilde	~
Logical Sum	∨
Extended Tilde	⁓
Backslash	\
Slash	/
Hollow Dot	∘
Intersection	∩
Union	∪
Minus with Dot Over	∸

SIGNS OF COMPARISON, cont.

Five comparison signs were presented in the Preliminary Lesson.

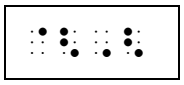
⠨⠅	Equals	=
⠨⠂	Greater Than	>
⠐⠅	Less Than	<
⠰⠆	Proportion	∷
⠐⠂	Ratio	:

5.6 More Comparison Signs

⠈⠣⠠⠣	Equivalence	≎
⠨⠨⠂	Greater Than with Curved Sides	≻
⠸⠇	Identity	≡
⠸⠐⠅	Inclusion	⊂
⠨⠐⠅	Less Than with Curved Sides	≺
⠈⠑	Membership	∈
		(also ε or Ɛ)
⠫⠇	Parallel to	∥
⠫⠏	Perpendicular to	⊥
⠠⠗	Relation	R
⠸⠨⠂	Reverse Inclusion	⊃
⠈⠢	Reverse Membership	∋
⠈⠱	Tilde, Simple	~
⠈⠠⠱	Tilde, Extended	∼∼
⠸⠿	Variation	∝
⠳	Vertical Bar	|

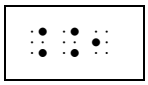
Examples of each symbol are shown in Sections 5.6.1-5.6.14. Note that, as with the other comparison signs you have learned, these signs are preceded and followed by a space regardless of the spacing shown in print.

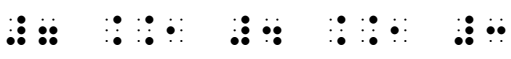
5.6.1 **Equivalence ("is equivalent to")**



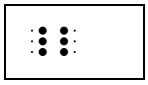
⋙ $x \fallingdotseq y$ 

5.6.2 **Greater Than with Curved Sides ("is greater than")**



⋙ $7 \succ 4 \succ 3$ 

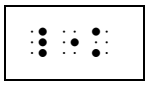
5.6.3 **Identity (Triple Bar)**



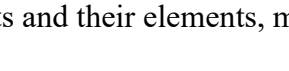
This symbol is used in several different contexts. Most commonly it means "is identical with" or "is congruent to". The transcriber uses the same symbol regardless of its meaning. Do not confuse the triple bar with the Greek letter Xi or the triple bond in Chemistry.

⋙ $A + B \equiv B + A$ 

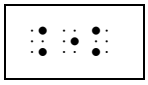
5.6.4 **Inclusion**



This symbol is used for sets and their elements, meaning "is contained in" or "is a subset of".

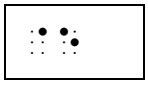
⋙ $A \subset D$ 

5.6.5 **Less Than with Curved Sides ("is less than")**



⋙ $5 \prec 9 \prec 11$ 

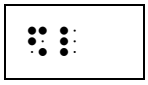
5.6.6 **Membership**



This symbol is used for sets and their elements, meaning "is an element of" or "belongs to". It must not be mistaken for the Greek uncapitalized epsilon even though it may be referred to as such.

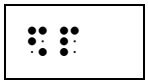
⋙ $5 \in B$

5.6.7 **Parallel To ("is parallel to")**



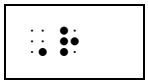
⋙ $AB \parallel CD$

5.6.8 **Perpendicular To ("is perpendicular to")**



⋙ $PQ \perp RS$

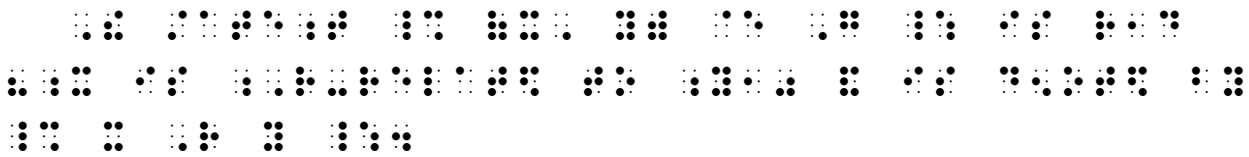
5.6.9 **Relation**



When a letter R is used between two expressions to show relation ("is related to"), the letter is treated as a sign of comparison with a space before and after it regardless of print spacing. Read the surrounding narrative to realize the meaning of the letter "R" in order to transcribe it correctly. Note that other letters or signs may also be used to show relation.

Example 5-23

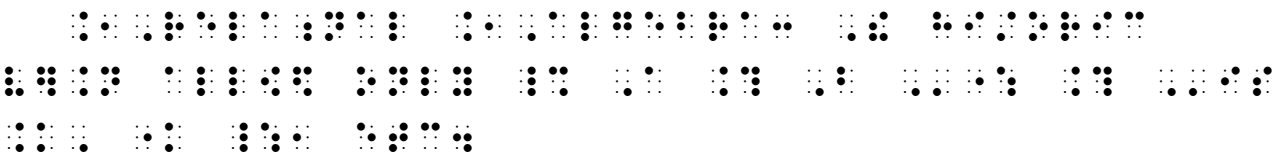
The statement $(x, y) \in G$ is read "x is R-related to y," and is denoted by xRy.



The letter R is functioning as a comparison sign here, so it is preceded and followed by a space in the transcription.

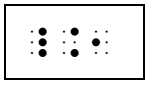
Example 5-24

Relational Algebra: The historic version allowed only $A\theta B$ where θ is $=$, $<$, etc.

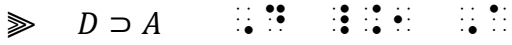


Greek letter theta is the relation symbol in this example. As a sign of comparison, it is preceded and followed by a space in the transcription.

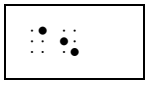
5.6.10 **Reverse Inclusion**



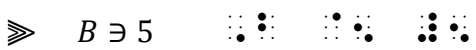
This symbol may mean "contains" or, in logic, "implies".



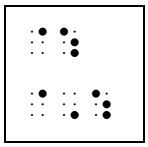
5.6.11 **Reverse Membership**



This symbol means "contains the element".



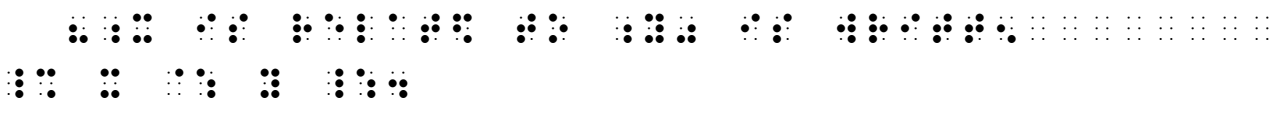
5.6.12 **Tilde and Extended Tilde**



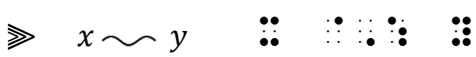
a. The simple tilde has one peak. When used as a comparison sign, the tilde means "is related to" or "is similar to".

Example 5-25

"x is related to y" is written $x \sim y$.



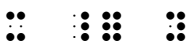
b. The extended tilde has more than one peak.



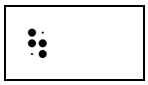
c. The tilde functioning as a sign of comparison is often encountered in the study of set theory and relations. The tilde is also used as a sign of operation. Consider the context to determine its meaning. (See Section 5.4.9.) When the tilde's meaning cannot be determined from context, follow print spacing.

5.6.13 Variation ("varies as")

$x \propto y$



5.6.14 Vertical Bar

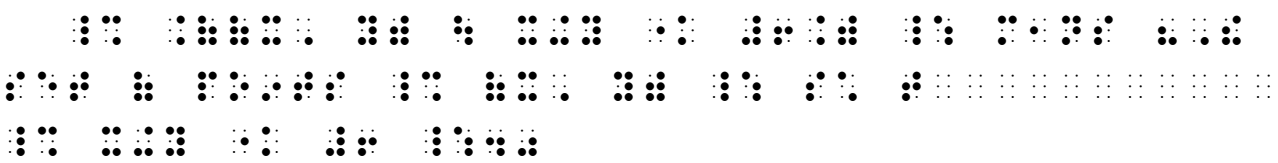


a. When used as a sign of comparison, the vertical bar means "such that." It usually occurs in an expression within braces. The print copy may or may not show the vertical bar as a spaced symbol. In braille, however, comparison signs must be preceded and followed by a space.

$\{x \in E | \Phi(x)\}$

Example 5-26

$\{(x, y) | x + y < 6\}$ means "The set of points (x, y) such that $x + y < 6$."



b. The vertical bar is also used as a sign of grouping (Lesson 2) and as an operation sign (Section 5.4.11). It is also used in probability notation (Lesson 13). Consider the context to determine its meaning and apply proper spacing according to its function. When the vertical bar's meaning cannot be determined from context, follow print spacing.

5.7 Special Case—A Colon Meaning "Such That"

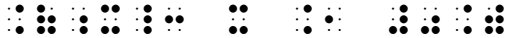
In set notation or mapping notation, a colon may be used to mean "such that". Although functioning as a sign of comparison, the Nemeth spacing rules governing comparison signs are not followed because a colon is a mark of punctuation.

a. Spacing in print may vary. Regardless of the spacing shown in print, in braille the colon is not preceded by a space. A punctuation indicator is required before the colon. Follow print for spacing after the colon.

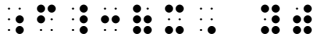
b. Follow Nemeth rules for a "single letter" for the letter on either side of the colon regarding use/nonuse of the English-letter indicator. (See Lessons 3 and 4.)

⋙ $m:W$ 

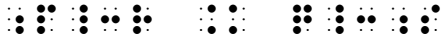
In the print copy, this colon is unspaced. Letter m needs an English-letter indicator because it is preceded by a space and is followed by punctuation. Letter W needs an English-letter indicator because it is preceded by punctuation and is followed by a space.

⋙ $\{x : x > 0\}$ 

In the print copy, this colon is preceded and followed by a space. The first x needs an English-letter indicator because it is touching only one grouping sign and is followed by punctuation. The second x does not need an English-letter indicator because it is followed by a comparison sign.

⋙ $f:(x, y)$ 

In the print copy, this colon is unspaced. Letter f needs an English-letter indicator because it is preceded by a space and is followed by punctuation. Letters x and y do not need an English-letter indicator because they are in an enclosed list.

⋙ $p:r = q:s$ 

In the print copy, these colons are unspaced. Letter p needs an English-letter indicator because it is preceded by a space and is followed by punctuation. Letters r and q do not need an English-letter indicator because they each are next to a comparison sign. Letter s needs an English-letter indicator because it is preceded by punctuation and is followed by a space.

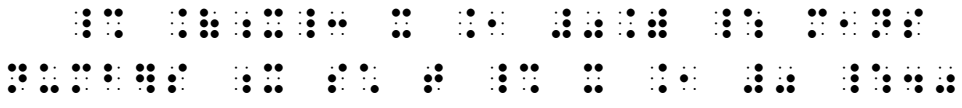
Example 5-27

$m:W$ means "the mapping m of W."

In the print copy, this colon is unspaced.

Example 5-28

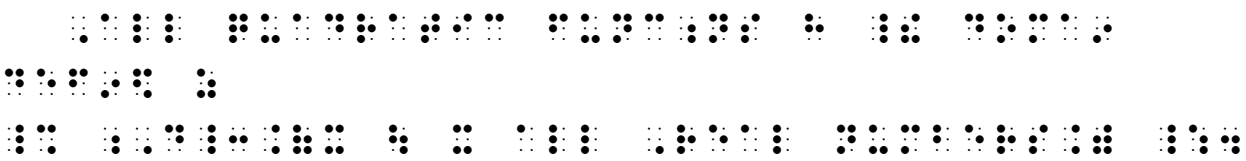
$\{x : x > 0\}$ means "The set of numbers x such that $x > 0$."



In the print copy, this colon is preceded and followed by a space.

Example 5-29

All quadratic functions have their domain defined as $D: \{x | x$ all Real numbers$\}$.



In the print copy, this colon is unspaced. An English-letter indicator is not needed for either letter x because they are next to a sign of comparison—the vertical bar.

c. Do not mistake the "such that" colon for a ratio symbol, which looks like a colon in print. Section P9 of the Preliminary Lesson discusses the ratio symbol.

PRACTICE 5C

Instructions: Review the spacing rules for comparison signs before transcribing the practice. Assume all tildes and vertical bars are comparison signs in these expressions.

$A \ni x$

$\{x \in A \mid x \backsim a\}$

$j \eqcirc k$

$r \propto s$

$f(x) \equiv D(x) \cdot q(x)$

$-12 \prec -4 \prec 0$

$.9 \succ .5$

$(A \cup E) \subset (F \cup B)$

$Q \supset Z$

$PQR \sim P'Q'R'$

$AB \parallel \mathrm{MN}$

$CD \perp OP$

$EF \parallel GH \perp QR$

$\{m \mid 3(m-6) = -9\}$

$\{x : x \text{ has the property } T\}$

5.8 Signs of Comparison Compounded Vertically

When two or more simple signs of comparison are arranged one under the other, the combination becomes a single comparison sign compounded vertically. The symbol for the uppermost sign is written first, immediately followed by and unspaced from the symbol for the lower sign. Comparison signs compounded vertically not shown in this section are transcribed in accordance with this principle.

5.8.1 Greater Than or Equal To

	Bar Over Greater Than	⩾ or ⩾
	Equals Sign Over Greater Than	⋛ or ⪌
	Bar Under Greater Than	≥ or ⩾
	Equals Sign Under Greater Than	≧ or ⪌

The "equal to" sign may be printed as an equals sign or as a single line – either a horizontal bar or an oblique line. Note that both the horizontal bar and the oblique line are represented by the same braille symbol (dots 156).

⪌ $a \geqslant b$ 

⪌ $a \gtreqless b$ 

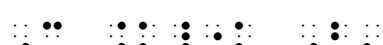
⪌ $x \geq y$

⪌ $|x| \geqq 0$

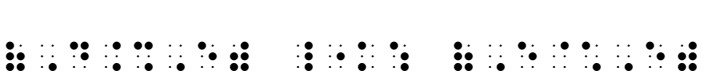
5.8.2 Inclusion ("is a subset of")

	Bar Over Inclusion	⊆
	Equals Sign Over Inclusion	⫅
	Bar Under Inclusion	⊆
	Equals Sign Under Inclusion	⫅

⪌ $C \subseteq B'$ 

⪌ $C \subseteqq B'$ 

⪌ $(D \cap E) \subseteq (E \times E)$



⫸ $(D \cap E) \subseteqq (E \times E)$

⠷⠠⠙⠨⠩⠠⠑⠾ ⠸⠐⠅⠨⠅ ⠷⠠⠑⠈⠡⠠⠑⠾

5.8.3 Intersection

⠨⠩⠱	Bar Under Intersection	$\underline{\cap}$
⠨⠩⠨⠅	Equals Sign Under Intersection	$\underline{\underline{\cap}}$

a. The intersection sign is a sign of comparison when modified by a bar or equals sign below it. It is also called a "cap".

⫸ $X \underline{\cap} Y$ ⠠⠭ ⠨⠩⠱ ⠠⠽

⫸ $X \underline{\underline{\cap}} Y$ ⠠⠭ ⠨⠩⠨⠅ ⠠⠽

b. An unmodified intersection sign is a sign of operation. See Section 5.4.4.

5.8.4 Less Than or Equal To

⠱⠐⠅	Bar Over Less Than	$\overline{<}$ or ⋜
⠨⠅⠐⠅	Equals Sign Over Less Than	$\overline{\overline{<}}$ or ⪙
⠐⠅⠱	Bar Under Less Than	$\leq$ or $\leqslant$
⠐⠅⠨⠅	Equals Sign Under Less Than	$\leqq$ or ⪕

⫸ $v - 1 \overline{<} 5$ ⠧⠤⠂ ⠱⠐⠅ ⠼⠢

⫸ $v - 1$ ⪙ 5 ⠧⠤⠂ ⠨⠅⠐⠅ ⠼⠢

⫸ $6 \leq x \leq 9$ ⠼⠖ ⠐⠅⠱ ⠭ ⠐⠅⠱ ⠼⠔

⫸ 6 ⪕ x ⪕ 9 ⠼⠖ ⠐⠅⠨⠅ ⠭ ⠐⠅⠨⠅ ⠼⠔

5.8.5 Logical Product

⠱⠈⠩	Bar Over Logical Product	$\overline{\wedge}$
⠱⠈⠩⠱	Bar Over and Bar Under Logical Product	$\underline{\overline{\wedge}}$
⠱⠈⠩⠨⠅	Bar Over and Equals Sign Under Logical Product	$\underset{=}{\overline{\wedge}}$
⠈⠩⠱	Bar Under Logical Product	$\underline{\wedge}$
⠨⠅⠈⠩	Equals Sign Over Logical Product	$\overset{=}{\wedge}$
⠨⠅⠈⠩⠱	Equals Sign Over and Bar Under Logical Product	$\underline{\overset{=}{\wedge}}$
⠨⠅⠈⠩⠨⠅	Equals Sign Over and Equals Sign Under Logical Product	$\underset{=}{\overset{=}{\wedge}}$
⠈⠩⠨⠅	Equals Sign Under Logical Product	$\underset{=}{\wedge}$

a. The logical product sign is a sign of comparison meaning "meet" when modified by a bar or equals sign above or below it.

⪢ $ABD \ \overline{\wedge}\ A'B'D'$

⠠⠁⠠⠃⠠⠙ ⠱⠈⠩ ⠠⠁⠄⠠⠃⠄⠠⠙⠄

⪢ $\{A\} \underset{=}{\wedge} K$ ⠨⠷⠠⠁⠨⠾ ⠈⠩⠨⠅ ⠠⠅

⪢ $p \underline{\wedge} q$ ⠏ ⠈⠩⠱ ⠟

b. An unmodified logical product sign is a sign of operation. See Section 5.4.5.

5.8.6 Logical Sum

⠱⠈⠬	Bar Over Logical Sum	$\overline{\vee}$
⠱⠈⠬⠱	Bar Over and Bar Under Logical Sum	$\underline{\overline{\vee}}$
⠱⠈⠬⠨⠅	Bar Over and Equals Sign Under Logical Sum	$\underset{=}{\overline{\vee}}$
⠈⠬⠱	Bar Under Logical Sum	$\underline{\vee}$
⠨⠅⠈⠬	Equals Sign Over Logical Sum	$\overset{=}{\vee}$
⠨⠅⠈⠬⠱	Equals Sign Over and Bar Under Logical Sum	$\underline{\overset{=}{\vee}}$
⠨⠅⠈⠬⠨⠅	Equals Sign Over and Equals Sign Under Logical Sum	$\underset{=}{\overset{=}{\vee}}$
⠈⠬⠨⠅	Equals Sign Under Logical Sum	$\underset{=}{\vee}$

a. The logical sum sign is a sign of comparison meaning "join" when modified by a bar or equals sign above or below it.

⫸ $ABC\ \overline{\vee}\ A'B'C'$ ⠠⠁⠠⠃⠠⠉ ⠱⠈⠬ ⠠⠁⠄⠠⠃⠄⠠⠉⠄

⫸ $P\left(E\ \underline{\vee}\ F\right)$ ⠠⠏⠷⠠⠑ ⠈⠬⠱ ⠠⠋⠾

b. An unmodified logical sum sign is a sign of operation. See Section 5.4.6.

5.8.7 Reverse Inclusion

⠱⠸⠨⠂	Bar Over Reverse Inclusion	$\overline{\supset}$
⠨⠅⠸⠨⠂	Equals Sign Over Reverse Inclusion	$\overset{=}{\supset}$
⠸⠨⠂⠱	Bar Under Reverse Inclusion	$\underline{\supset}$
⠸⠨⠂⠨⠅	Equals Sign Under Reverse Inclusion	$\underset{=}{\supset}$

⫸ $B\ \overline{\supset}\ A$ ⠠⠃ ⠱⠸⠨⠂ ⠠⠁

⫸ $D\ \underset{=}{\supset}\ C$ ⠠⠙ ⠸⠨⠂⠨⠅ ⠠⠉

5.8.8 Tilde

⠱⠈⠱	Bar Over Single Tilde	$\overset{-}{\sim}$
⠨⠅⠈⠱	Equals Sign Over Single Tilde	$\overset{=}{\sim}$
⠈⠱⠈⠱	Double Tilde	$\approx$
⠱⠈⠱⠈⠱	Bar Over Double Tilde	$\overset{-}{\approx}$
⠨⠅⠈⠱⠈⠱	Equals Sign Over Double Tilde	$\overset{=}{\approx}$
⠈⠱⠱	Bar Under Single Tilde	$\simeq$
⠈⠱⠨⠅	Equals Sign Under Single Tilde	$\cong$
⠈⠱⠈⠱⠱	Bar Under Double Tilde	$\underset{-}{\approx}$
⠈⠱⠈⠱⠨⠅	Equals Sign Under Double Tilde	$\underset{=}{\approx}$

⫸ $3.14159 \approx 3.1416$

⠼⠒⠨⠂⠲⠂⠢⠔ ⠈⠱⠈⠱ ⠼⠒⠨⠂⠲⠂⠖

⫸ $ABC \cong DEF$

⠠⠁⠠⠃⠠⠉ ⠈⠱⠨⠅ ⠠⠙⠠⠑⠠⠋

5.8.9 Union

⠨⠬⠱	Bar Under Union	$\underline{\cup}$
⠨⠬⠨⠅	Equals Sign Under Union	$\underset{=}{\cup}$

a. The union sign is a sign of comparison when modified by a bar or equals sign above or below it. It may also be referred to as a "cup".

⫸ $A \underline{\cup} B$ ⠠⠁ ⠨⠬⠱ ⠠⠃

b. An unmodified union sign is a sign of operation. See Section 5.4.10.

PRACTICE 5D

Instructions: Transcribe "Signs of Comparison Compounded Vertically" as a cell-5 heading. Change the two-column format to a nested list by starting each phrase in cell 1, with each math expression starting on a new line in cell 3. *Code Switching Guidelines:* To maintain clarity within the nested list layout, place the opening Nemeth Code indicator at the end of the UEB phrase, and place the Nemeth Code terminator on the same line with the related math.

Signs of Comparison Compounded Vertically

Greater Than or Equal To	$ab \eqslantgtr de$
	$\|y\| \geq 0$
Less Than or Equal To	$q - 7 \eqslantless 5z$
	$-6 \leq x \leq -1$
Inclusion and Reverse Inclusion	$C' \subset F'$ and $D \supset C$
	$(B \cap E) \subseteq (E \times E)$
Intersection and Union ("Cup")	$X \underline{\cap} Y$
	$X \underline{\underline{\cap}} Y$
	$A \underline{\cup} B$
Logical Product and Logical Sum	$QRS \barwedge Q'R'S'$
	$y \underline{\wedge} z$ and $M(E \underline{\vee} H)$
	$ABC \barvee A'B'C'$
Tilde	$3.14159 \approx 3.1416$
	$ABC \cong DEF$

5.9 Signs of Comparison Compounded Horizontally

When two or more signs of comparison are arranged side by side, the combination becomes a single comparison sign compounded horizontally. A multipurpose indicator (dot 5) is inserted between the unspaced symbols to indicate that they are printed horizontally, not vertically. Comparison signs compounded horizontally not shown in this section are transcribed in accordance with this principle.

5.9.1 Greater Than ...

⠸⠣⠜	Greater Than Followed by Less Than	> <
⠸⠣⠨⠅⠜	Greater Than Followed by Equals Followed by Less Than	> = <

⋙ $n > < 1$ ⠸ ⠸⠣⠜⠣⠜ ⠸⠣

⋙ $n > = < 1$ ⠸ ⠸⠣⠨⠅⠜⠣⠜ ⠸⠣

5.9.2 Less Than ...

⠸⠜⠣	Less Than Followed by Greater Than	< >
⠸⠜⠨⠅⠣	Less Than Followed by Equals Followed by Greater Than	< = >

⋙ $n < > 1$ ⠸ ⠸⠜⠣⠜⠣ ⠸⠣

⋙ $n < = > 1$ ⠸ ⠸⠜⠨⠅⠣⠜⠣ ⠸⠣

5.10 Negated Signs of Comparison

In print, a sign of comparison may be negated by a vertical or a slanted line drawn through it. The print negation symbol may be slanted in either direction. In braille, ⠠ represents any of the print negation lines. ⠠ is placed immediately before the sign of comparison being negated. Some samples are shown in the box below. Negated signs of comparison not illustrated here are transcribed according to the same principle.

⠠⠶	Negated Equals Sign	≠ or ≠
⠠⠸⠇	Negated Parallel To	∦
⠠⠸⠪	Negated Perpendicular To	∠̸
⠠⠣⠨⠅	Negated "Bar Under Greater Than"	⩲
⠠⠐⠑	Negated Membership	∉ or ∉

⋙ $4 \times 13 \neq 14$ ⠼⠲⠈⠡⠂⠒ ⠌⠨⠅ ⠼⠂⠲

⋙ $4 \ngeq 7$ ⠼⠲ ⠌⠨⠂⠱ ⠼⠶

⋙ $9 \notin D$ ⠼⠔ ⠌⠈⠑ ⠠⠙

⋙ CD ∦ EF ∠̸ GH ⠠⠉⠠⠙ ⠌⠫⠇ ⠠⠑⠠⠋ ⠌⠫⠏ ⠠⠛⠠⠓

PRACTICE 5E

Instructions: Use the principles learned in Section 5.10 to construct the negation symbols that are not shown in the sample box.

(1) $x \notin$ A means "x is not an element of A".

(2) By typing <=, the symbol ≤ will appear. By typing /< , the symbol ≮ will appear.

(3) A ⊈ B means that at least one element of A is not an element of B.

(4) WXY ≁ VXW

(5) Since L ∦ M and M ∦ N, does it follow that L ∦ N?

(6) The domain is all $x \neq -4, 0, 5$.

Format: Nemeth Instructions

5.11 Margins for Instructions Preceding Itemized Material (5-3)

The Nemeth code makes a distinction regarding instructions preceding a set of itemized problems. Following a blank line, the instructions begin in cell 5 with runovers in cell 3. The related itemized material begins on the next line.

Exceptions: (a) Instructions may begin on line 1 if no running head is used. (b) When instructions immediately follow a cell-5 or a cell-7 heading, the blank line before the instructions is unnecessary. (c) If the itemized material itself requires a leading blank line, such as for spatial material (to be studied later in this course), a blank line is inserted.

The print document may use a distinctive typeform for instructions. Typeform used solely as a visual enhancement is disregarded in the braille transcription, according to UEB and *Braille Formats* guidelines.

In Example 5-30, the dashed line indicates a print page turn.

Example 5-30

Problem Set 7F Tell whether the following ratios are equivalent.

1. $3 : 2 = 75 : 50$
2. $6 : 4 = 15 : 30$

Which math sentence is true? Which is false?

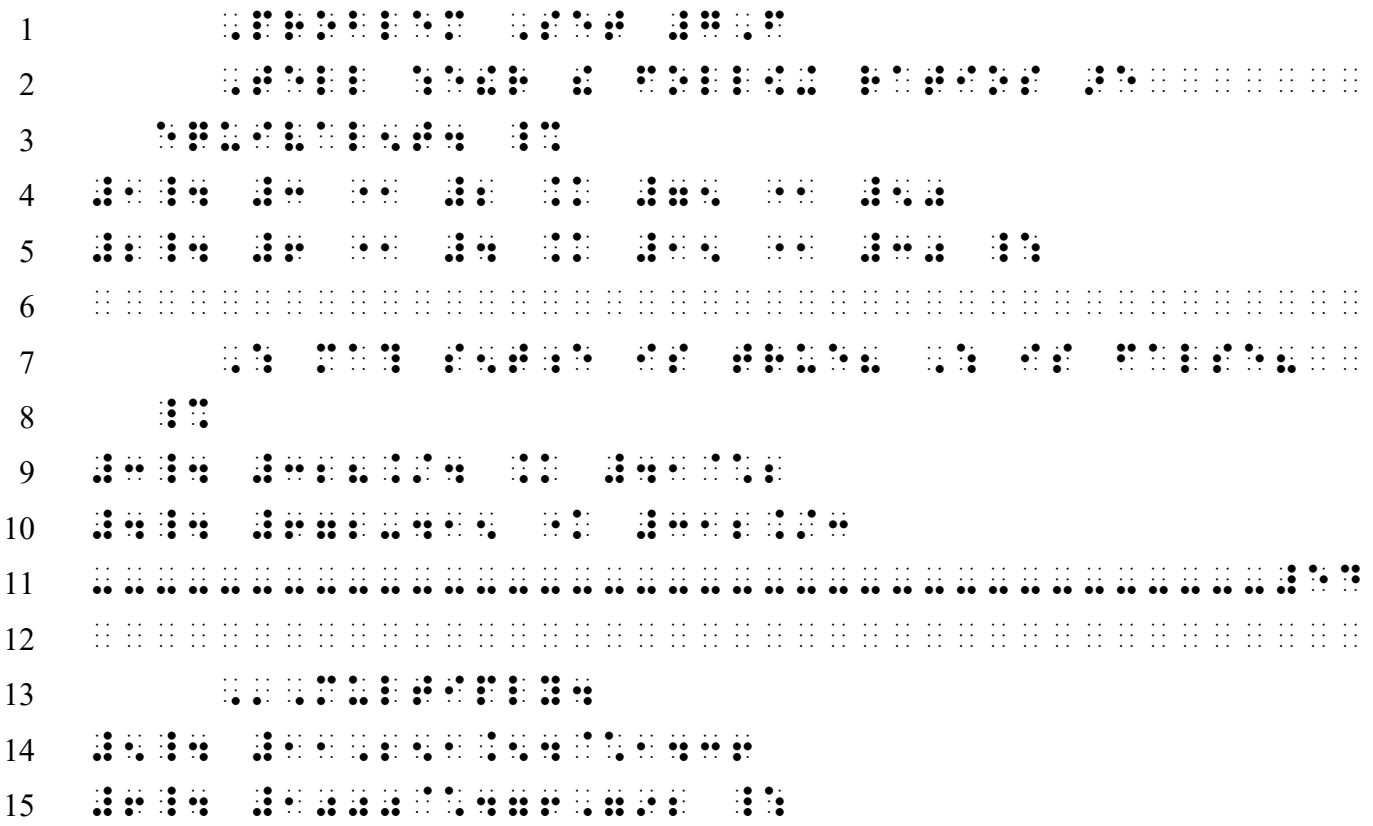
3. $328 \div 4 = 41 \times 2$
4. $672 - 415 < 312 \div 3$

— —

54

Multiply.

5. $11{,}251.54 \times 1436$
6. $1000 \times 476{,}792$



Line 1: Cell-5 heading
Lines 2-3: Instructions (5-3) immediately following a cell-5 heading
Lines 4-5: Itemized material following instructions
Line 6: A blank line precedes instructions
Line 7: Instructions begin in cell 5. The opening switch does not fit on this line.
Line 8: The opening Nemeth Code indicator is in the runover cell of the instructions (cell 3).
Lines 9-10: Itemized material following instructions
Line 12: Blank line precedes instructions
Line 13: Instructions (5-3, no runover)
Lines 14-15: Itemized material following instructions

5.11.1 **Keep Together.** An exercise set consists of instructions followed by related itemized problems. If an exercise set will fit on one page, it should not be divided between braille pages. If an exercise set requires more than one braille page, *Braille Formats* states that the instructions (directions) should be on the same braille page as at least one of the questions or problems to which they apply.

5.11.2 **Code Switching and Instructions.** The opening Nemeth Code indicator may be placed after the last word of the instructions. If there is no room on the line, place the switch indicator in the runover cell of the instructions (cell 3). (An exception applies to spatially arranged material, which will be covered in Lesson 9.)

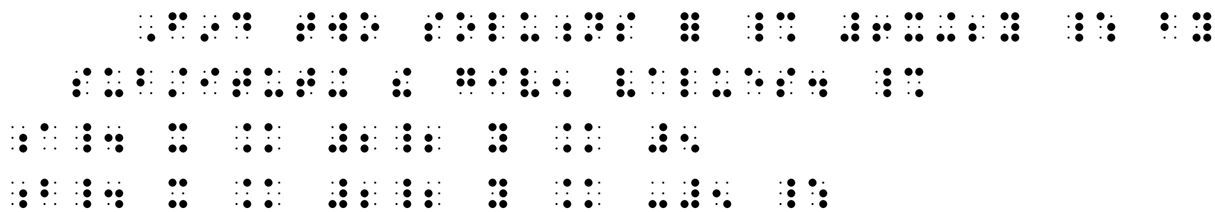
If instructions end with a Nemeth expression and the subsequent math problem starts with a Nemeth expression, Nemeth may be left in effect between the end of the instructions and the start of the problem.

Example 5-31

Find two solutions for $6x + 2y$ by substituting the given values.

a. $x = 2; y = 5$

b. $x = 2; y = -5$

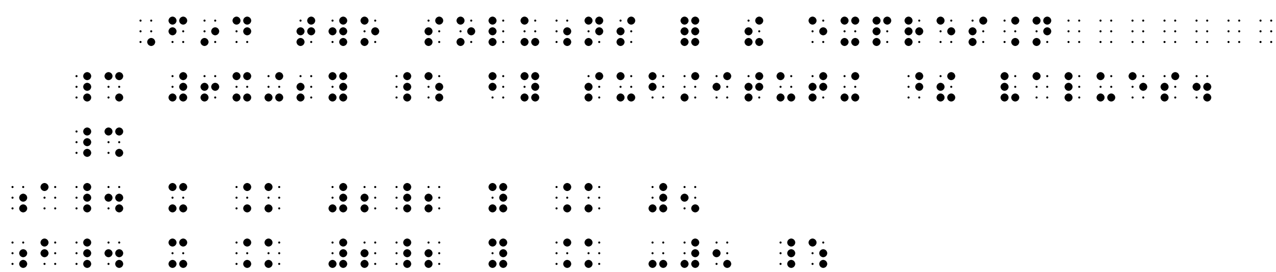


Example 5-32

Find two solutions for the expression $6x + 2y$ by substituting these values.

a. $x = 2; y = 5$

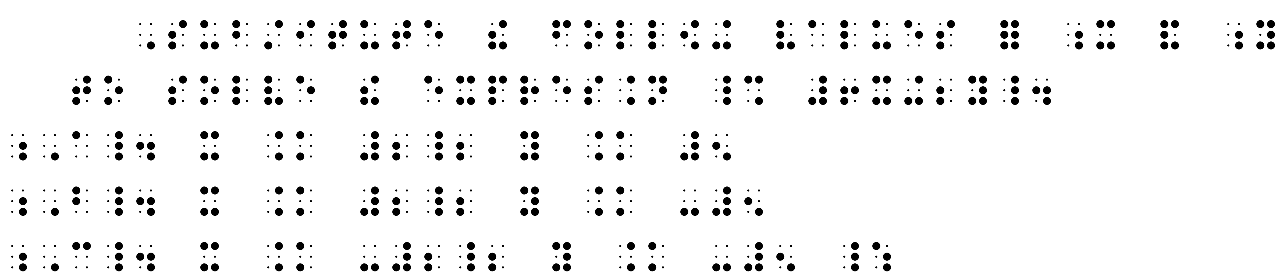
b. $x = 2; y = -5$



Example 5-33

Substitute the following values for x and y to solve the expression $6x + 2y$.

A. $x = 2; y = 5$ B. $x = 2; y = -5$ C. $x = -2; y = -5$



Reminder: When the print copy arranges itemized material side by side across the page and there are no subdivisions, Nemeth format rules dictate that all identifiers start in cell 1.

5.12 Narrative Directions

"Instruction" format (5-3) applies only to text that is followed by a set of numbered or lettered problems. If the material following the text is not numbered or lettered, if the directions themselves are numbered or lettered, or if the narrative is not giving explicit directions that apply to the following itemized material, then other established formats are followed as illustrated in this section. We may refer to such text as "directions".

5.12.1 **Margins for Directions Preceding Unitemized Material (3-1).** If there is no numbered or lettered exercise material following the directions, the text is treated as narrative material and is transcribed as a (3-1) paragraph. The (5-5) style of "directions" given in *Braille Formats* does not apply in a transcription that contains Nemeth.

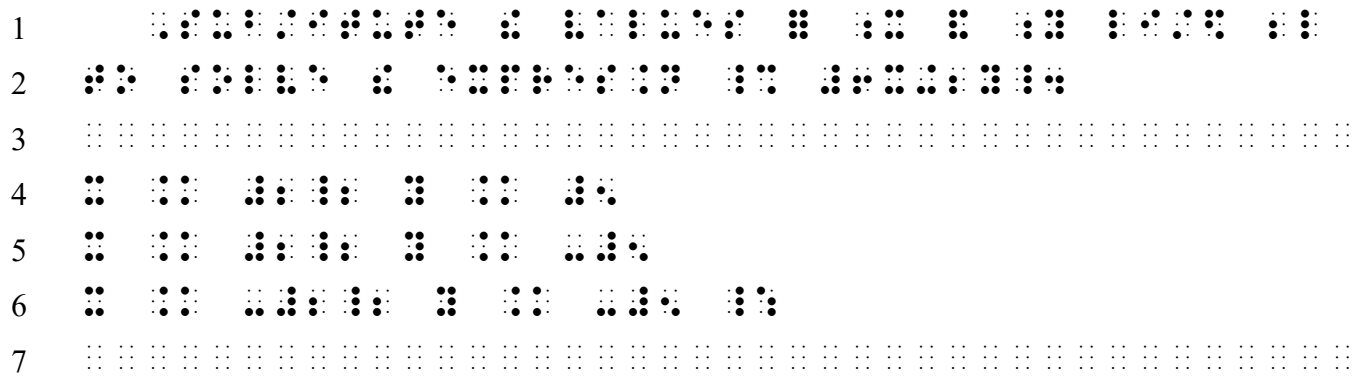
Example 5-34

Substitute the values for x and y listed below to solve the expression $6x + 2y$.

$x = 2; y = 5$

$x = 2; y = -5$

$x = -2; y = -5$

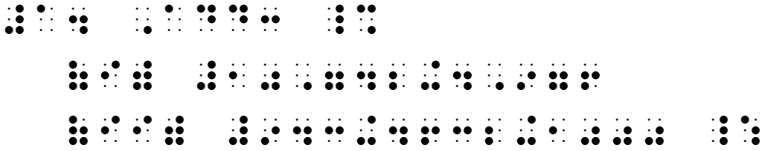


5.12.2 **Itemized Directions.** Directions that are preceded by a number or letter are formatted according to the Nemeth margin rules for itemized material.

Example 5-35

1. Add:

(i) $10{,}742 + 4{,}976$ (ii) $943 + 4632 + 1000$

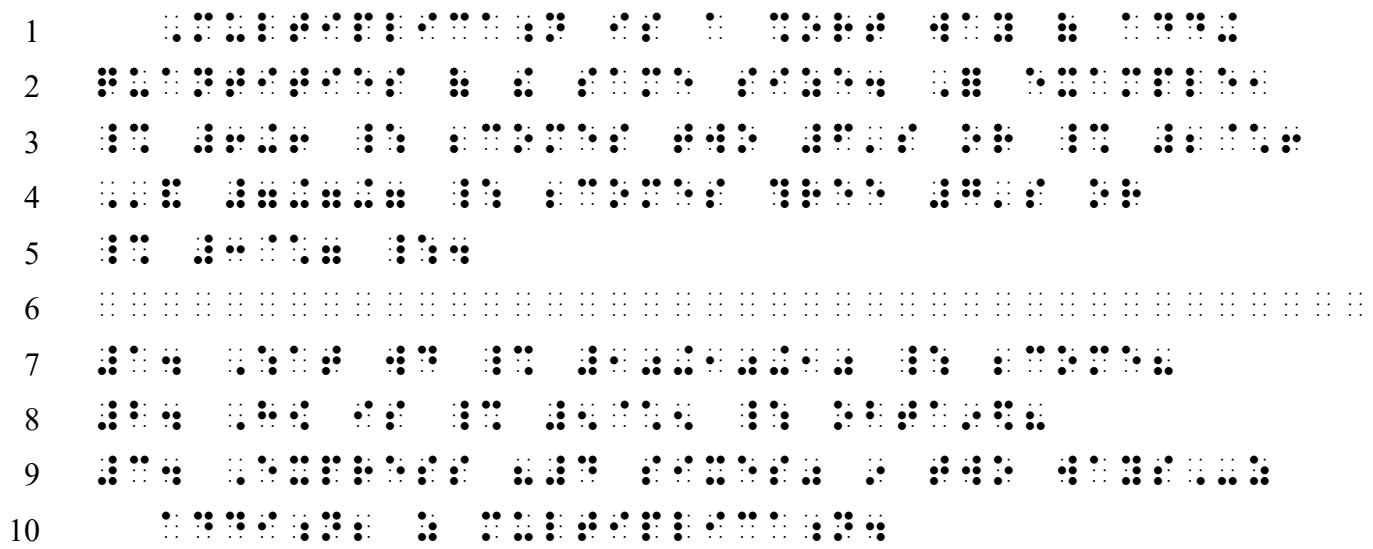


5.12.3 **Margins for Narrative Preceding Itemized Material (3-1).** Only explicit textual matter is formatted as (5-3) "instructions". If the narrative is purely explanatory, regular paragraphing is applied, and a blank line precedes the itemized material.

Example 5-36

Multiplication is a short way of adding quantities of the same size. For example, 6 + 6 becomes two 6's or 2 × 6 and 7 + 7 + 7 becomes three 7's or 3 × 7.

1. What would 10 + 10 + 10 become?
2. How is 5 × 5 obtained?
3. Express "4 sixes" in two ways—as addition; as multiplication.



PRACTICE 5F

Instructions: Treat "Signs of Comparison" and "Adding and Subtracting Integers" as cell-5 headings.

Signs of Comparison

These examples illustrate the basic spacing rules for comparison signs learned in this unit.

(1) $5 \prec 9 \prec 11$

(2) $11.7 \succ 1.17$

(3) $550 : 11 :: \ ? : 12$

Adding and Subtracting Integers

Find the sum or difference as indicated by the signs.

1) $-6 + -5 =$ ____

2) $5 + -19 =$ ____

3) $-7 - -13 =$ ____

4) $29 - -24 =$ ____

For further practice, see Addendum 1—Reading Practice.

Submit Exercise 5 to your instructor.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 5A

1 ⠸⠩

2 ⠼⠲⠬⠤⠂⠠ ⠼⠲⠴⠴⠬⠤⠂⠴⠠ ⠼⠖⠤⠬⠂⠠ ⠼⠖⠴⠴⠤⠬⠂⠴

3 ⠨⠍⠬⠤⠂⠨⠖⠲⠢⠨⠎

4 ⠼⠢⠴⠤⠐⠬⠢ ⠨⠅ ⠼⠲⠢

5 ⠼⠢⠴⠬⠐⠤⠢ ⠨⠅ ⠿

6 ⠤⠼⠒⠤⠐⠤⠒ ⠨⠅ ⠼⠴

7 ⠠⠁⠸⠯⠠⠃ ⠨⠅ ⠠⠃⠸⠯⠠⠁

8 ⠁⠈⠼⠷⠃⠈⠼⠉⠾ ⠨⠅ ⠷⠁⠈⠼⠃⠾⠈⠼⠉

9 ⠷⠂⠬⠆⠾⠈⠼⠷⠒⠬⠲⠾ ⠨⠅ ⠼⠒⠈⠼⠼⠶

10 ⠨⠼⠠⠁ ⠨⠅ ⠨⠼⠠⠃

11 ⠼⠨⠢⠨⠼⠼⠨⠔ ⠨⠅ ⠼⠨⠔⠨⠼⠼⠨⠢

12 ⠈⠷⠷⠏⠸⠻⠏⠾⠸⠻⠷⠟⠸⠻⠟⠾⠈⠾

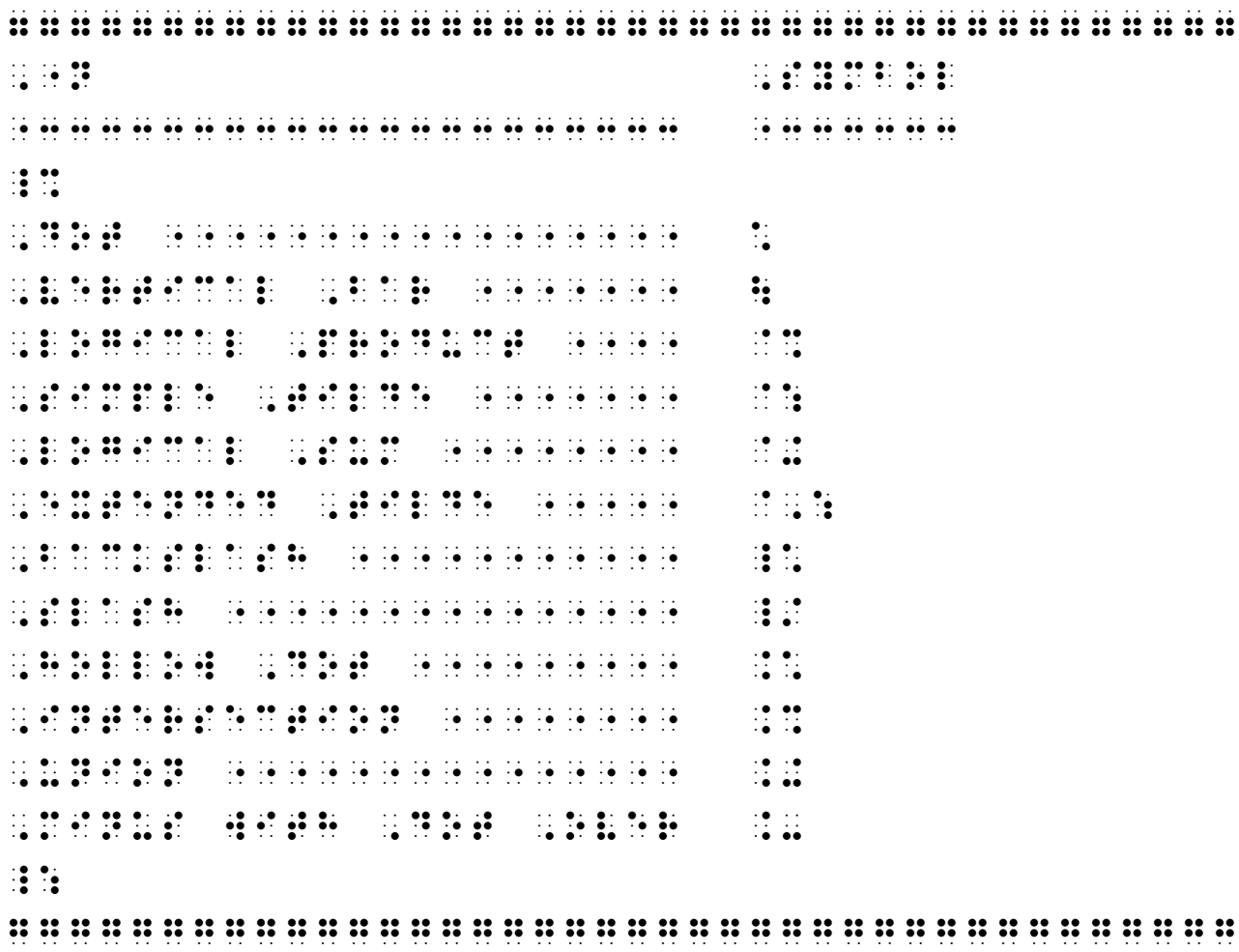
13 ⠎⠈⠠⠏⠞ ⠨⠅ ⠥⠈⠠⠏⠧

14 ⠼⠂⠈⠠⠏⠼⠒ ⠨⠅ ⠼⠲⠈⠠⠏⠼⠒

15 ⠍⠈⠠⠎⠽ ⠨⠅ ⠽⠈⠠⠎⠍ ⠨⠅ ⠽

16 ⠼⠢⠈⠠⠎⠼⠖ ⠨⠅ ⠼⠖⠈⠠⠎⠼⠢ ⠨⠅ ⠼⠖ ⠸⠱

PRACTICE 5B



PRACTICE 5C

1 ⠸⠩

2 ⠠⠁ ⠈⠢ ⠭

3 ⠨⠷⠭ ⠈⠑ ⠠⠁ ⠣ ⠭ ⠈⠱ ⠁⠨⠾

4 ⠚ ⠈⠣⠠⠣ ⠅

5 ⠗ ⠸⠿ ⠎

6 ⠋⠷⠭⠾ ⠸⠇ ⠠⠙⠷⠭⠾⠡⠟⠷⠭⠾

7 ⠤⠼⠂⠆ ⠨⠐⠅ ⠤⠼⠲ ⠨⠐⠅ ⠼⠴

8 ⠼⠨⠔ ⠨⠨⠂ ⠼⠨⠢

9 ⠷⠠⠁⠨⠬⠠⠑⠾ ⠸⠐⠅ ⠷⠠⠋⠨⠬⠠⠃⠾

10 ⠠⠟ ⠸⠨⠂ ⠠⠵

11 ⠠⠏⠠⠟⠠⠗ ⠈⠱ ⠠⠏⠄⠠⠟⠄⠠⠗⠄

12 ⠠⠁⠠⠃ ⠫⠇ ⠠⠍⠠⠝

13 ⠠⠉⠠⠙ ⠫⠏ ⠠⠕⠠⠏

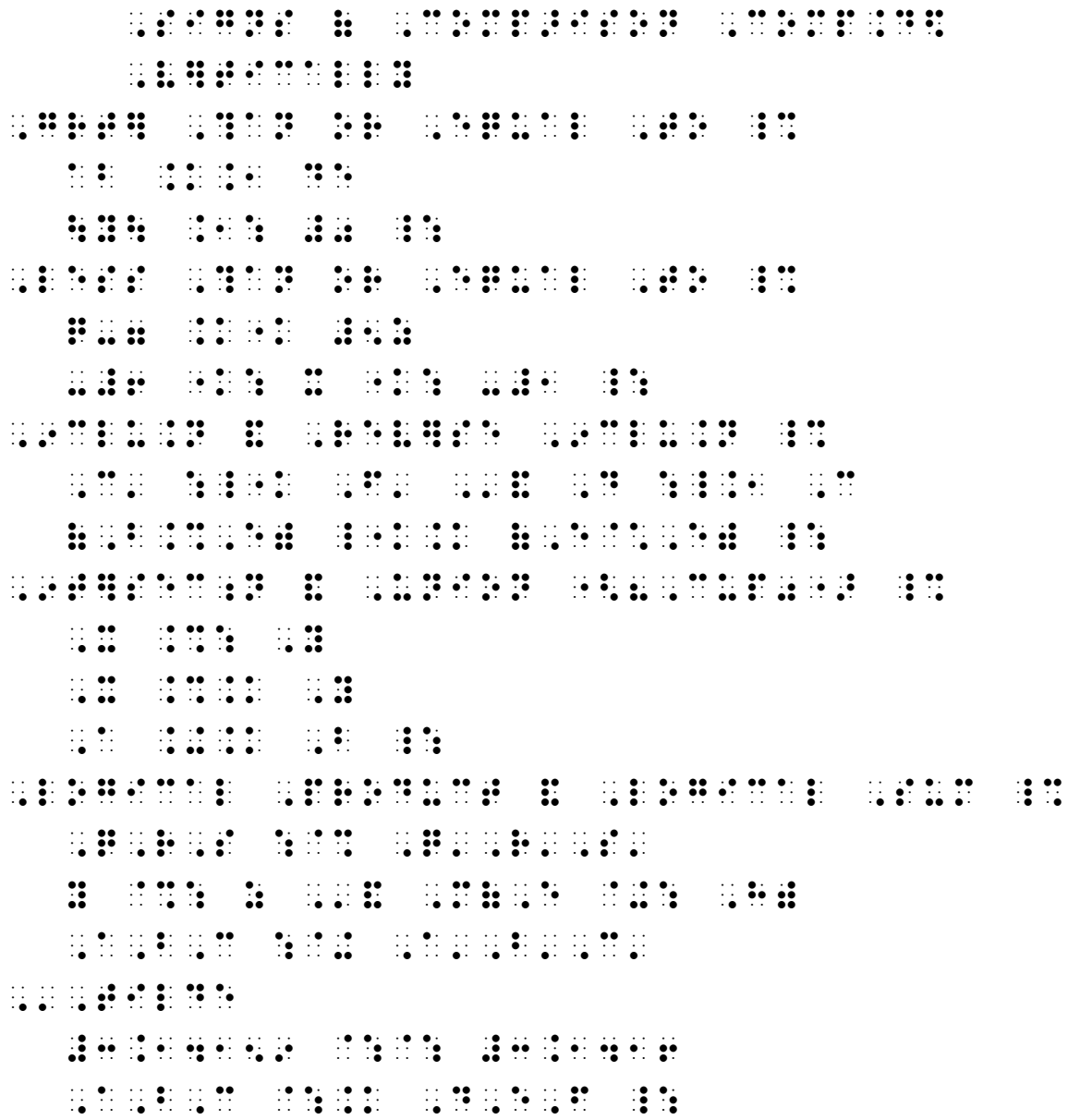
14 ⠠⠑⠠⠋ ⠫⠇ ⠠⠛⠠⠓ ⠫⠏ ⠠⠟⠠⠗

15 ⠨⠷⠍ ⠳ ⠼⠒⠷⠍⠤⠖⠾ ⠨⠅ ⠤⠼⠔⠨⠾

16 ⠨⠷⠰⠭⠸⠒ ⠰⠭ ⠓⠁⠎ ⠞⠓⠑ ⠏⠗⠕⠏⠑⠗⠞⠽ ⠰⠠⠞⠨⠶ ⠸⠱

Notes regarding the last item: The words are part of the math and so are transcribed uncontracted without switching out of Nemeth. Each single letter requires an English-letter indicator because each is preceded and followed by a space and/or punctuation (or begins a new line). The presence of a single grouping symbol is ignored when determining whether an English-letter indicator is needed.

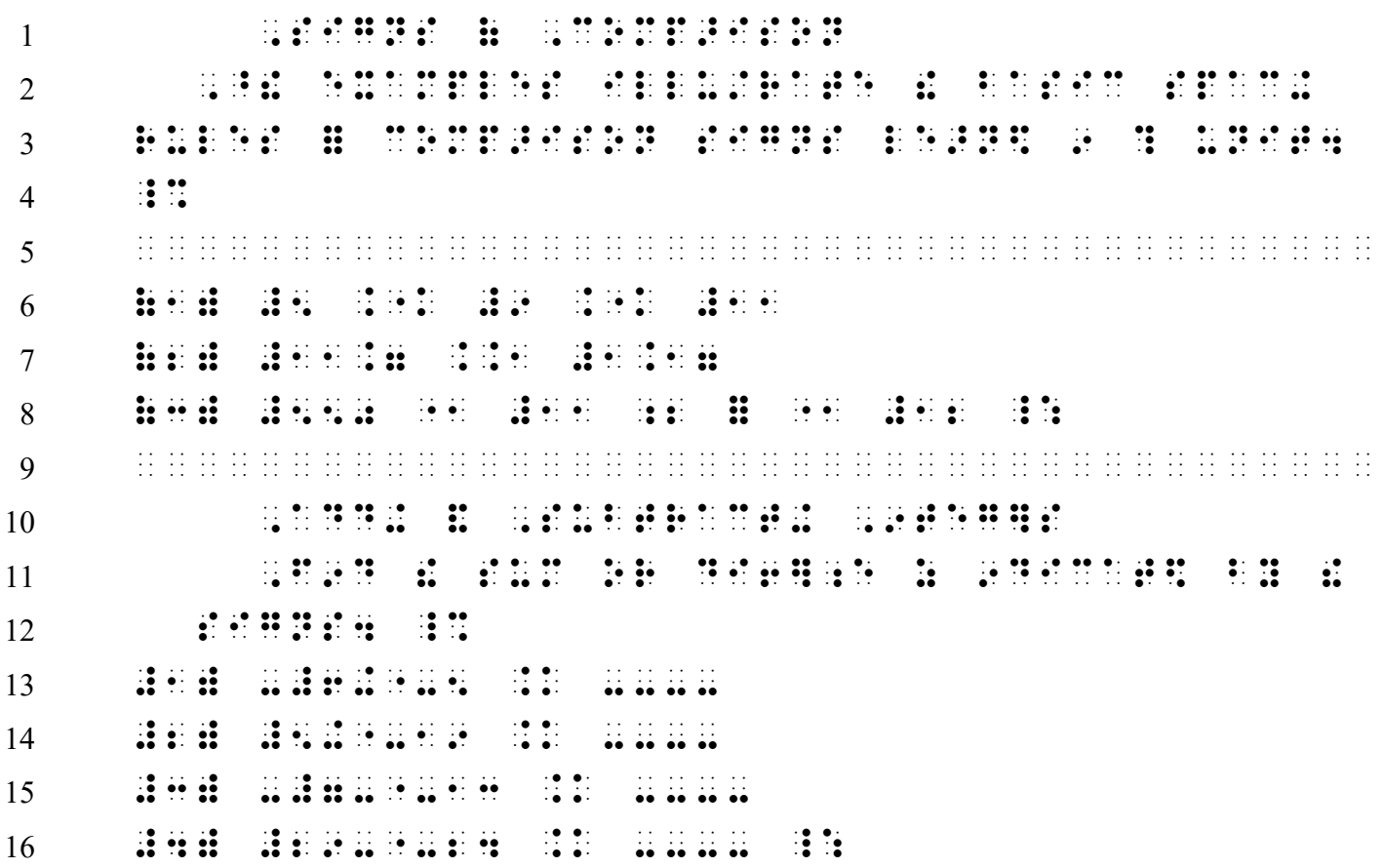
PRACTICE 5D



PRACTICE 5E

⠐⠣⠼⠁⠐⠜ ⠸⠩ ⠭ ⠌⠈⠑ ⠠⠁ ⠸⠱ ⠍⠂⠝⠎ ⠦⠰⠭ ⠊⠎ ⠝ ⠁⠝
⠑⠇⠑⠰⠞ ⠷ ⠠⠁⠴⠲
⠐⠣⠼⠃⠐⠜ ⠠⠃⠽ ⠞⠽⠏⠬ ⠸⠩ ⠐⠅⠐⠨⠅ ⠸⠱⠂ ⠮ ⠎⠽⠍⠃⠕⠇
⠸⠩ ⠐⠅⠱ ⠸⠱ ⠺ ⠁⠏⠏⠑⠜⠲ ⠠⠃⠽ ⠞⠽⠏⠬
⠸⠩ ⠸⠌⠐⠐⠅ ⠸⠱⠂ ⠮ ⠎⠽⠍⠃⠕⠇ ⠸⠩ ⠌⠐⠅ ⠸⠱ ⠺
⠁⠏⠏⠑⠜⠲
⠐⠣⠼⠉⠐⠜ ⠸⠩ ⠠⠁ ⠌⠸⠐⠅⠱ ⠠⠃ ⠸⠱ ⠍⠂⠝⠎ ⠞ ⠁⠞ ⠇⠂⠌
⠐⠕ ⠑⠇⠑⠰⠞ ⠷ ⠠⠁ ⠊⠎ ⠝ ⠁⠝ ⠑⠇⠑⠰⠞ ⠷ ⠰⠠⠃⠲ ⠸⠩
⠷⠲⠾ ⠠⠺⠠⠭⠠⠽ ⠌⠈⠱ ⠠⠧⠠⠭⠠⠺
⠷⠢⠾ ⠠⠄⠠⠎⠔⠉⠑ ⠠⠇ ⠌⠫⠇ ⠠⠍ ⠠⠄⠯ ⠠⠍ ⠌⠫⠇ ⠠⠝ ⠸⠱⠂
⠙⠕⠑⠎ ⠭ ⠋⠕⠇⠇⠪ ⠞ ⠸⠩ ⠠⠇ ⠌⠫⠇ ⠠⠝ ⠸⠱⠦
⠐⠣⠼⠋⠐⠜ ⠠⠮ ⠙⠕⠍⠁⠔ ⠊⠎ ⠁⠇⠇
⠸⠩ ⠭ ⠌⠈⠁ ⠤⠼⠲⠠ ⠼⠴⠠ ⠼⠢ ⠸⠱⠲

PRACTICE 5F



Lines 2-4: 3-1 narrative paragraph.

Lines 11-12: 5-3 Nemeth instructions. No blank line between cell-5 heading and Nemeth instructions. See Section 5.11(b).