

LESSON 8

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LESSON PREVIEW

Fractions, mixed numbers, and radical expressions are studied. The "linked expression" is defined, and its format is discussed in detail, including the special case of the nested linked expression.

Note: The font size in some of the examples is larger than standard, for clarity. Some of the larger symbols may appear to be bold. Do not consider the size difference or the darker image to be a variant typeform.

INTRODUCTION TO FRACTIONS

Two types of fractions are presented in this lesson: simple fractions (including mixed numbers) and complex fractions.

A fraction is composed of three parts: a numerator, a denominator, and a fraction line.

$$\begin{array}{ccc}
 \text{numerator} \rightarrow & & \\
 & \frac{3}{4} & \leftarrow \text{fraction line} \\
 \text{denominator} \rightarrow & &
 \end{array}$$

Fractions are printed in a variety of ways. The numerator may be printed above the denominator, or they may be printed on the same level. The fraction line may be horizontal or diagonal. Here are three examples of the fraction "three fourths" printed in different styles.

$$\frac{3}{4} \qquad 3/4 \qquad 3/4$$

The numerator and/or denominator may also consist of or contain words or abbreviations. Here are four examples.

$$\text{m/s} \qquad (\textit{Spoken: meters per second})$$

$$\text{ft./sec.} \qquad (\textit{Spoken: feet per second})$$

$$\frac{\text{rise}}{\text{run}} \qquad (\textit{Spoken: rise over run})$$

$$3.5\%/year \qquad (\textit{Spoken: 3.5 percent per year})$$

In a technical transcription, fractions are transcribed in Nemeth.

Simple Fractions

[NC 13.1]

8.1 Definition

For the purposes of the Nemeth code, a simple fraction is one whose numerator and denominator contain no fractions, except possibly at the superscript or subscript level. These are simple fractions:

$$\frac{1}{2} \quad \frac{a^2}{b^2} \quad \frac{72 \text{ mi.}}{4 \text{ hr.}} \quad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}} \quad 5/12$$

This is not a simple fraction because the numerator contains a fraction: $\frac{1/3}{5}$

PRACTICE 8B

Instructions: Determine whether the slash is used mathematically, that is, does it require a switch to Nemeth? If it does, then determine if fraction indicators are required.

Diagonal Simple Fraction Line

- A) How many $\frac{2}{3}$'s are there in $\frac{5}{6}$?
 - B) Energy is absorbed at the rate of 880 J/s for each square meter of the surface.
 - C) $y(0) = \pi/4$
 - D) $1 \text{ ft/sec} \approx 0.6818 \text{ mph}$
 - E) In $y \frac{1}{5}$, y is the coefficient of the fraction $1/5$.
 - F) True/False: The rise/run ("rise over run") ratio is 5 in graph A.
 - G) $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
 - H) A 5-year CD went from earning interest at the rate of 12.06%/year in 1984 to earning less than 0.87%/year in 2015.
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Complex Fractions

[NC 13.5]

8.6 Definition of Complex Fraction

A complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers.

$$\begin{array}{l} \text{numerator} \\ \text{denominator} \end{array} \quad \frac{4 \frac{3}{4}}{5} \quad \leftarrow \text{complex fraction line}$$

Here are more samples of complex fractions.

$$\frac{\frac{4}{3}}{12} \quad \frac{\frac{a}{b} - \frac{c}{d}}{\frac{a}{b} + \frac{c}{d}} \quad \frac{1}{3/8} \quad \frac{1/3}{2/4}$$

Reminder: A fraction is not a complex fraction if the only fractions it contains are at the superscript or subscript level. Such a fraction is a simple fraction.

This is a simple fraction, not a complex fraction: $\frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

8.6.1 Use of Complex Fraction Indicators and Complex Fraction Lines. The opening and closing complex fraction indicators are used to enclose a complex fraction.

Complex Fraction Indicators

⠠⠠ Opening

⠠⠠ Closing

The main complex fraction line is represented by its appropriate braille symbol—either horizontal or diagonal.

⠠⠠ Horizontal Complex Fraction Line —

⠠⠠ Diagonal Complex Fraction Line /

PRACTICE 8D

Instructions: Begin each complex fraction on a new braille line, not side by side as printed. Read left to right.

Complex Fractions

$$\frac{\frac{1}{8} + \frac{3}{4}}{7}$$

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} - \frac{7}{9}}$$

$$\frac{1/3+1/4}{4/5-1/2}$$

$$\frac{\frac{\pi}{8}}{2\pi}$$

$$\frac{\frac{a}{b}}{c}$$

$$\frac{33\frac{1}{3}}{100}$$

$$\frac{3/5}{6}$$

$$\frac{3}{5} / \frac{7}{9}$$

PRACTICE 8E

Instructions: Determine the formatting before beginning your transcription. Where does each paragraph begin? Which expressions are embedded and which are displayed? What is the proper cell placement for the displayed expressions?

These are simple fractions:

$$\frac{1}{2} \quad \frac{a^2}{b^2} \quad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$$

This is not a simple fraction: $\frac{1/3}{2/3}$

Review the rules in Section 6.12.4 in Lesson 6 regarding an ellipsis on the baseline of writing when it follows a superscript.

$$x^{\frac{1}{2}} \dots x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}} \dots \frac{x^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1}$$

Plot the points $\left(-\frac{1}{2}, 4\right)$, $\left(3, 4\frac{1}{4}\right)$, and $\left(-9, \frac{3}{4}\right)$. Then express $\frac{a^{3/4}}{b^{5/4}}$ in radical form.

8.13 Spacing

The spacing before and after a radical expression is subject to the spacing rules for the signs preceding or following the radical expression.

$$\gg \sqrt{9} - \sqrt{4} = 1$$

No space is left between a radical expression and a letter, a numeral, a fraction, a sign of grouping, a braille indicator, or another radical expression when these items are unspaced in print and belong to the same expression.

$$\gg \sqrt{5}y$$

$$\gg \sqrt{x^2}$$

$$\gg 2a\sqrt{4ab}$$

$$\gg \sqrt{4}\sqrt{87} = 2\sqrt{87}$$

$$\gg \sqrt{y} dx + (1 + x) dy = 0, y(0) = 1$$

$$\sqrt{y} dx + (1 + x) dy = 0, y(0) = 1$$

Reminder: In print, derivative notation dx , dy , etc. is often preceded and followed by a space within an expression, for clarity. In braille, the terms are not spaced unless a space is required with the item preceding or following them. (Review Section 4.15.1 in Lesson 4.)

Example 8-29

Simplify. $\frac{2 - \sqrt{\frac{1}{4}}}{3 - \sqrt{\frac{1}{2}}}$

$$\frac{2 - \sqrt{\frac{1}{4}}}{3 - \sqrt{\frac{1}{2}}}$$

8.14 Index of Radical

A small number or letter that may appear next to the radical sign is the *index* of the radical. This print sample shows an index "3".

$$\sqrt[3]{27}$$

In braille, the index-of-radical indicator and the index precede the radical sign.

8.19 Enclosed Lists with Radical Expressions

Radical expressions may be part of an enclosed list.

$$\gg (\sqrt{9}, 3, \sqrt{4}, 2\sqrt{6})$$



PRACTICE 8G

Nested Radical Expressions

$$(1) \sqrt{-\frac{1}{2} - i\frac{\sqrt{3}}{2}}$$

$$(2) \sqrt{\sqrt{13} + \sqrt{15} + \sqrt{117}}$$

$$(3) \sqrt{1 - \sqrt{a-b}} \times \sqrt{1 + \sqrt{a-b}}$$

$$(4) \sqrt{b^3\sqrt{b}\sqrt{b}}$$

$$(5) \sqrt[3]{b\sqrt{c}\sqrt{abc}}$$

$$(6) \sqrt{a^2}\sqrt{b^4}\sqrt{c} = ab^2\sqrt{c}$$

$$(7) (s^2\sqrt[3]{s^4})^2$$

$$(8) \sqrt[3]{x^2\sqrt{64x^6}}$$

$$(9) \sqrt{\sqrt[3]{\sqrt[4]{\sqrt[5]{b^{48}}}}}$$

$$(10) \sqrt{x_1 + \sqrt{x_2}}$$

$$(11) q^{\sqrt{r}} + s$$

$$(12) \sqrt{c + d + e + \dots}$$

LINKED EXPRESSIONS

[NC 26.5]

8.20 Definition of Linked Expression

A linked expression contains at least one sign of comparison. The part preceding the first sign of comparison is called the *anchor*. Each remaining part, beginning with a sign of comparison and ending before the next sign of comparison, is called a *link*. In its simplest form, $x = y$ is a linked expression where x is the anchor and $= y$ is the link.

$$12.5\% > \frac{1}{10} \quad \text{The anchor is } 12.5\% \text{ and the link is } > \frac{1}{10}$$

$$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$$

The anchor is 6×245 , followed by three links each beginning with an equals sign.

8.21 Division of Linked Expressions

A linked expression is considered to be one unit and must not be divided between lines if it will fit on one braille line within the current margins. Observe the following points when an anchor and its link or links will not fit on one line.

- (1) If the entire linked expression will fit on one braille line, do not divide it. See [Example 8-31](#). Revisit Examples 1-20, 1-21, and 1-22 in Lesson 1.
- (2) If the entire linked expression will not fit on one braille line, use as much of the current line as possible before dividing. Then divide before the comparison sign of a link. See [Example 8-32](#) and [Example 8-33](#).
- (3) It is not necessary to divide at every link. See [Example 8-34](#). (*Exception*: If the linked expression is printed in the nested layout discussed in [Section 8.22](#), other rules apply.)
- (4) If the anchor or a link will not fit on one braille line, further rules apply. Those rules will be covered in Lesson 15.

Example 8-31

Follow the steps. $12.5\% = .125 = \frac{125}{1000} = \frac{1}{8}$.

Although the anchor and the first link will fit on line 1, since the entire linked expression fits on one line, the anchor begins on line 2.

PRACTICE 8H**Linked Expressions**

1. Is the following inequality true? $\frac{3}{5}\left(\frac{2}{3}x - \frac{1}{2}\right) > \frac{2}{5}\left(\frac{1}{4}x + \frac{1}{3}\right)$
 2. $33\frac{1}{3}\% + 40\% + 61\frac{2}{3}\% = 134\frac{3}{3}\% = 135\%$
 3. In multiplying $5\frac{3}{4} \times 46$, recall that $5\frac{3}{4} = 5 + \frac{3}{4}$. Therefore, $46 \times \left(5 + \frac{3}{4}\right) = (46 \times 5) + \left(46 \times \frac{3}{4}\right) = 264\frac{1}{2}$.
-

8.22 Special Case—Nested Linked Expressions [NC 26.5.3.b]

An expression with two or more links may be subject to special Nemeth format rules if it appears in print in a particular arrangement as described in this section.

Print Layout

- The expression is displayed.
 - There is an exception regarding itemized problems – see [Section 8.22.b](#).
- The first line contains only the anchor or only the anchor and the first link.
- Each following link begins on a new line, and the comparison signs beginning each link are vertically aligned.
 - An exception applies to the last line – see [Section 8.22.a](#).

The following linked expression meets the three requirements.

To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.

$$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c). \end{aligned}$$

- a. It is common for the last line of the expression to contain more than one link. As long as the other conditions are met, this layout meets the requirements for this nested format.

We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:

$$\begin{aligned} 12\frac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \frac{125}{1000} = \frac{1}{8} \end{aligned}$$

- b. The next sample shows an itemized problem with no narrative. Although the expression is not displayed, this arrangement follows all of the other print layout specifications: the comparison signs are vertically aligned, and – other than on the first and last line – no sign of comparison is preceded by any expression on its left. Rules regarding this layout are outlined in [Section 8.22.4](#).

$$\begin{array}{l}
 1. \quad 12\frac{1}{2}\% = 12.5\% \\
 \qquad \qquad = .125 \\
 \qquad \qquad = \frac{125}{1000} = \frac{1}{8}
 \end{array}$$

When the print layout meets the definition of a nested linked expression, one of the following Nemeth formats is applied.

8.22.1 Margin Requirements for a Nested Linked Expression. The margins which are applied to a nested linked expression follow a reliable pattern, which can be generalized as follows.

- The anchor begins two cells to the right of the runover margin of the material to which it is displayed.
- Each link begins on a new braille line, indented two cells to the right of the anchor.
- Runovers to anchor or links begin four cells to the right of the anchor cell.

Note: Rules regarding how to divide a link that will not fit on the line will be discussed in Lesson 15. In this lesson, in order to illustrate runovers within a nested linked expression, a runover line will begin with a sign of operation.

8.22.2 Nested Linked Expression Displayed to Narrative. When a nested linked expression occurs in unitemized explanatory portions of the text (3-1), the anchor begins in cell 3 and each link begins in cell 5. In braille, each link begins on a new line, even when the print copy shows more than one link on the last line.

Reminder: A line is not skipped above or below displayed mathematical material unless a blank line is required under other rules or guidelines.

PRACTICE 8I
Recognizing a Nested Linked Expression

To test the equation $R_t = \frac{R}{n}$, use four 1000- Ω resistors wired in series to predict a total resistance of 250 Ω .

$$R_t = \frac{R}{n} = \frac{1000 \Omega}{4}$$

$$\frac{1000 \Omega}{4} = 250 \Omega$$

Then, by using the resistance theory equation

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n},$$

plug the 150- Ω and 1000- Ω resistors into the equation as R_1 and R_2 .

$$\begin{aligned} \frac{1}{R_t} &= \frac{1}{150 \Omega} + \frac{1}{1000 \Omega} \\ &= 0.007 + 0.001 \\ &= 0.008 \\ R_t &= \frac{1}{0.008} = 125 \Omega \end{aligned}$$

8.22.3 **Nested Linked Expression Displayed to Itemized Material.** Apply the general pattern when a nested linked expression is displayed to itemized material: begin the anchor two cells to the right of the current runover margin; begin each link two cells to the right of the anchor cell; begin runovers four cells to the right of the anchor cell.

- a. **Itemized Text with No Subdivisions.** When a nested linked expression occurs in itemized text containing no subdivisions (1-3), the anchor begins in cell 5 and each link begins in cell 7. Runovers begin in cell 9.
- b. **Itemized Text with Subdivisions.** When a nested linked expression occurs in itemized text containing subdivisions (1-5; 3-5), the anchor begins in cell 7 and each link begins in cell 9. Runovers begin in cell 11.

PRACTICE 8J**Multiplying Mixed Numbers**

A. $2\frac{1}{2} \cdot 1\frac{1}{4} = \left(2 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{4}\right)$
 $= 2 + \frac{2}{4} + \frac{1}{2} + \frac{1}{8}$
 $= 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8}$
 $= 2 + 1 + \frac{1}{8} = 3\frac{1}{8}$

B. What will the remainder be in this problem?

$$\begin{aligned} 4\frac{1}{3} \times 3\frac{2}{5} \times 9\frac{11}{15} \times 2\frac{3}{4} \\ &= \frac{13}{3} \times \frac{17}{5} \times \frac{146}{15} \times \frac{11}{4} \\ &= \frac{13 \times 17 \times 146 \times 11}{3 \times 5 \times 15 \times 4} \\ &= \frac{354,926}{900} \\ &= 394 \text{ with a remainder of } \underline{\quad}. \end{aligned}$$

For further practice, see Addendum 1—Reading Practice.

Submit Exercise 8 to your instructor.

PRACTICE 8H

1
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

2
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

3
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

4
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

5
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

6
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

7
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

8
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

9
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

10
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

11
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

12
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

Lines 10-11: Notice that the print copy divided this equality after the equals sign, but the braille transcription follows Nemeth rules and divides before the comparison sign.

