

## LESSON 9

- [ARROWS](#)
  - [Construction of Braille Arrows](#)
  - [Vertical, Slanted, and Curved Arrow Shafts](#)
  - [Boldface and Compounded Arrows](#)
- [INTRODUCTION TO SPATIAL ARRANGEMENTS](#)
  - [Spatial Arrangements with Addition and Subtraction](#)
  - [Introduction to Cancellation](#)
  - [Arrangement on the Page](#)
  - [Placement of Code Switch Indicators](#)

[Answers to Practice Material](#)

### LESSON PREVIEW

The construction of braille arrows is demonstrated, showing several styles of arrowheads and arrow shafts. Braille format for spatially arranged addition and subtraction problems is studied.

*Note:* Due to the amount of vertical space used with spatial arrangements, this lesson consumes more pages than most.

## ARROWS

### [NC Rule 22]

#### 9.1 Arrows Used in Mathematics

Here are some samples of arrows which bear mathematical meaning.

$$A \rightarrow B \qquad \overleftrightarrow{PQ} \qquad 2\text{CO} \rightleftharpoons \text{CO}_2 + \text{C}$$

$$f: a \mapsto b \qquad P \uparrow Q \qquad A \rightarrow B \leftrightarrow C \leftarrow D$$

Note that some chemical arrows are constructed differently from math arrows. In a chemistry transcription, the arrows of *Chemical Notation Using the Nemeth Braille Code* should be used.

#### 9.2 Construction of Braille Arrows

The components of an arrow are transcribed in the following order.

- (1) The shape indicator.
- (2) The arrow direction if it must be indicated (vertical or slanted). (Sections [9.9.1](#) and [9.9.2](#).)
- (3) The typeface of the arrow if it must be indicated. ([Section 9.10](#).)
- (4) The left arrowhead or dot, if any. (Sections [9.5](#), [9.7](#), and [9.8](#).)
- (5) The arrow shaft, if required. (Sections [9.4](#) and [9.9](#).)
- (6) The right arrowhead or dot, if any. (Sections [9.4](#), [9.7](#), and [9.8](#).)

An explicit terminator is not necessary.



#### 9.3 Spacing and Punctuation with Arrows

When arrows function as a sign of comparison, the entire construction is spaced according to the rules governing comparison signs. When it is not functioning as a sign of comparison, an arrow is spaced according to its context. Spacing issues will be illustrated throughout this lesson.

Punctuation associated with an arrow is mathematical.

#### 9.4 Horizontal Arrow Shafts

The length of a shaft is indicated by the number of times the arrow shaft symbol is used. Two braille symbols represent the ordinary shaft length. By comparison, one braille symbol represents a short shaft, and three or more symbols indicate a longer shaft.

Horizontal Arrow Shafts		
⋯⋯	Ordinary length, single	—
⋯	Short, single	—
⋯⋯⋯	Long, single	—
⋯⋯	Ordinary length, double	==
⋯	Short, double	=
⋯⋯⋯	Long, double	==
⋯⋯⋯	Dashed	- - -
⋯⋯⋯	Dotted	...
⋯⋯⋯	Wavy	~

## 9.5 Barbed and Solid Arrowheads

An arrowhead may be barbed, solid, blunted, straight, or curved, and may occur at the left, at the right, or at both ends of an arrow shaft. An arrowhead may also appear with only its upper or lower barb. First, we will look at barbed and solid arrowheads appearing at the end of a horizontal arrow shaft.

*The short line in the barbed images represents the point where the arrow shaft meets the arrowhead.*

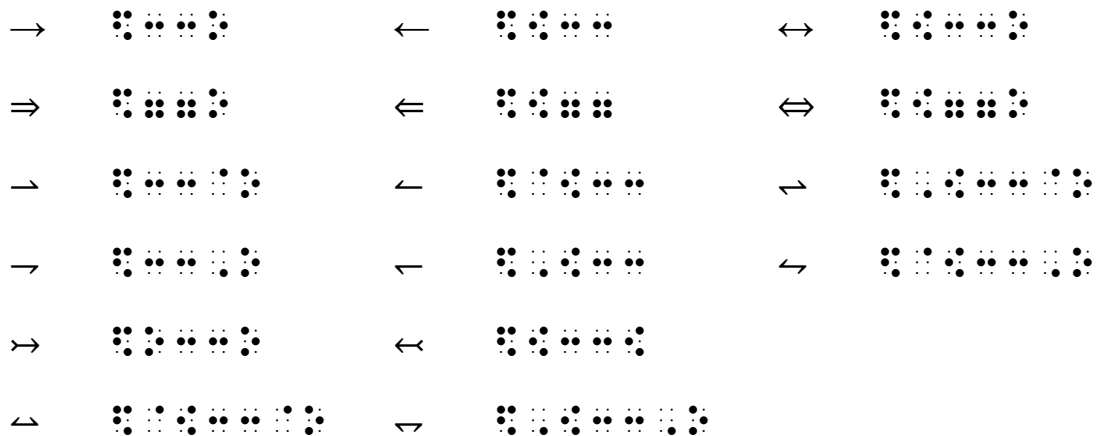
Barbed and Solid Arrowheads		
⋯	Barbed left full	←
⋯	Solid left	◄
⋯	Barbed right full	→
⋯	Solid right	►
⋯⋯	Barbed left lower	↙
⋯⋯	Barbed right lower	↘
⋯⋯	Barbed left upper	↖
⋯⋯	Barbed right upper	↗

Notice that solid arrowheads are transcribed using the same symbol as full barbed arrowheads.

Many types of arrows can be constructed using the shaft and arrowhead symbols. Some samples are shown on the next page.

Note: Code switch indicators are omitted in the isolated examples in this section.

Consider the shaft in the barbed arrows shown below to be the standard length for the book. A two-cell "ordinary length" shaft is transcribed. Notice the similarity between the print and braille symbols.



An additional rule may apply to the first arrow shown above. See [Section 9.6](#).

By comparison, the shafts in the arrows shown below are longer. A three-cell shaft is transcribed.



And, by comparison, the shafts in the arrows shown next are shorter. A one-cell shaft is transcribed.



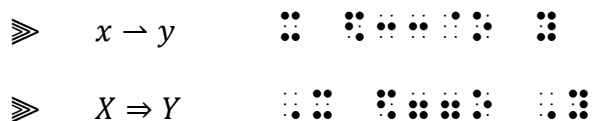
Other shaft styles are illustrated below.

Dashed and dotted

Wavy



a. **Spacing.** Note the spacing before and after these arrows which are functioning as comparison signs.





9.6.1 **Nonuse of the Contracted Form of the Right-Pointing Arrow.** All other right-pointing arrows require the use of the appropriate shaft symbol.

➤	$X \dashrightarrow Y$	⠠	⠠	⠠	⠠
➤	$X \rightsquigarrow Y$	⠠	⠠	⠠	⠠
➤	$X \longrightarrow Y$	⠠	⠠	⠠	⠠
➤	$X \rightarrow Y$	⠠	⠠	⠠	⠠

### PRACTICE 9A

*Instructions:* Leave one space (one blank cell) between the arrows in A. and B. Assume shafts in items A, C, and D to be of regular length, and in item B to be long. In item D, divide the long expression before the arrow that comes between the double parentheses. Assume all to be in regular type even though they may appear darker.

#### Arrows with Barbed Ends

- A.  $\Leftrightarrow$     $\rightarrow$     $\leftrightarrow$     $\longleftrightarrow$     $\dashrightarrow$     $\dashleftarrow$     $\rightsquigarrow$     $\leftarrow$
- B.  $\longleftarrow$     $\longleftrightarrow$     $\rightleftarrows$     $\Leftrightarrow$
- C. The principal operator of the left formula in Fig. 7.2 is  $\leftrightarrow$ , while the principal operator of the right formulas is  $\rightarrow$ .
- D. Construct a truth table for  $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ .

## 9.7 Blunted, Straight, and Curved Arrowheads

Here are three other types of arrowheads identified in the Nemeth code. Samples with a single shaft of ordinary length are shown. The longer horizontal line in each image represents the end of the arrow shaft.

*Please disregard the differences in size and line weight in these images.*

**BLUNTED ARROWHEADS**

Each blunted arrowhead can be left or right pointing.

⠠	Full	⠠⠠⠠⠠	┌
		⠠⠠⠠⠠	┐
		⠠⠠⠠⠠⠠	┌┐
⠠⠠	Lower only	⠠⠠⠠⠠	┌
		⠠⠠⠠⠠⠠	┐
⠠⠠	Upper only	⠠⠠⠠⠠	└
		⠠⠠⠠⠠⠠	┘

**STRAIGHT ARROWHEADS**

Each straight arrowhead can be left or right pointing.

⠠	Full	⠠⠠⠠⠠	├
		⠠⠠⠠⠠	┤
		⠠⠠⠠⠠⠠	├├
⠠⠠	Lower only	⠠⠠⠠⠠	├
		⠠⠠⠠⠠⠠	┤
⠠⠠	Upper only	⠠⠠⠠⠠	└
		⠠⠠⠠⠠⠠	┘

















- 9.9.3 **Curved Arrow Shaft [NC 22.5.1]** Direction indicators are not used with curved arrows. Instead, the curved shaft symbol is used, and the type of arrowhead reveals the direction of curvature.

 Curved Shaft

Rotation direction is indicated as follows. A curved shaft preceded by a *left*-pointing arrowhead represents a clockwise arrow. A curved shaft followed by a *right*-pointing arrowhead represents a counterclockwise arrow.

 Clockwise Curve			
 Counterclockwise Curve			

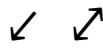
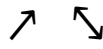
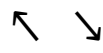
## PRACTICE 9C

### Other Types of Arrow Shafts

Vertical



Slanted



Curved



Can you figure out how to construct this spear with a northwest, blunted arrowhead?



## ***Boldface and Compounded Arrows***

### **9.10 Boldface Arrow [NC 22.6]**

When an arrow is printed in boldface type and the typeform is determined to be significant and therefore retained, the boldface typeform indicator (dots 456) is incorporated into the structure of the arrow. The typeform indicator is placed at the beginning of the arrow symbol before the arrowhead or arrow shaft. When a direction indicator is required, the direction indicator is transcribed first, followed by the boldface typeform indicator.

⠠⠠⠠⠠⠠	Boldface Pointing Right	➔
⠠⠠⠠⠠⠠	Boldface Pointing Left	➜
⠠⠠⠠⠠⠠⠠	Boldface Pointing Left and Right	↔
⠠⠠⠠⠠⠠⠠	Boldface Pointing Up	⬆
⠠⠠⠠⠠⠠⠠	Boldface Pointing Down	⬇
⠠⠠⠠⠠⠠⠠	Boldface Pointing Up and Down	↕

*Reminder: A right-pointing arrow in nonregular type requires a shaft. The contracted form is not used.*

### **9.11 Arrows Used as Signs of Comparison Compounded Vertically [NC 21.9]**

Lesson 5 introduced signs of comparison compounded vertically—two or more simple signs of comparison arranged one under the other in print. When arrows are so arranged, the same principle applies. The combination of symbols becomes a single comparison sign "compounded vertically". The uppermost arrow symbol is transcribed first, immediately followed by and unspaced from the symbol for the lower arrow. Each arrow requires a shape indicator.

When the arrow is part of a sign of comparison compounded vertically, the uncontracted form of the right-pointing arrow is used. The contracted form of the right-pointing arrow is never used in the construction of arrows compounded vertically.

⠠⠠⠠⠠⠠	Uncontracted Right-Pointing Arrow	➔
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Arrows compounded vertically are most often encountered in the subject of chemistry. When the rules of *Chemical Notation Using the Nemeth Braille Code* are being followed, use the arrow constructions from that code, which differ from the Nemeth symbols.

When arrows compounded vertically are encountered in mathematics, use the constructions shown in the box on the next page. Mathematical arrows which are not shown in this list should be transcribed in accordance with the principle outlined in this section.

	Pointing Right Over Pointing Left	
	Pointing Left Over Pointing Right	
	Long Pointing Right Over Short Pointing Left	
	Short Pointing Right Over Long Pointing Left	
	Pointing Right, Upper Barb Only Over Pointing Left, Lower Barb Only	
	Pointing Right Over Boldface Pointing Left	
	Boldface Pointing Left Over Pointing Right	
	Boldface Pointing Right Over Boldface Pointing Left	

## 9.12 Arrows Used as Signs of Comparison Compounded Horizontally [NC 21.11]

Lesson 5 also introduced the topic of signs of comparison compounded horizontally. When arrows are arranged side by side, the same principle applies. The combination of symbols becomes a single comparison sign "compounded horizontally". A multipurpose indicator (dot 5) is inserted between the unspaced symbols to indicate that the symbols are printed side by side. Arrows not shown in this list should be transcribed in accordance with this principle.

	Pointing Up Followed by Pointing Down	
	Pointing Down Followed by Pointing Up	
	Pointing Up Followed by Boldface Pointing Down	
	Boldface Pointing Down Followed by Boldface Pointing Up	

Arrows compounded horizontally are most often encountered in the subject of chemistry. When the rules of *Chemical Notation Using the Nemeth Braille Code* are being followed, use the arrow constructions from that code, which differ from the Nemeth symbols.

### 9.13 Nonmathematical Arrows

Nemeth arrows should not be used for nonmathematical purposes. Rules and guidelines in BANA publications examine flowchart arrows, arrows in graphic organizer diagrams, lead lines in tactile graphics, etc. Refer to *Guidelines and Standards for Tactile Graphics* for techniques regarding arrows in specific applications such as box-and-whisker plots, clock faces, graphs, line plots, measurement tools, number lines, Cartesian graphs, and spinners.

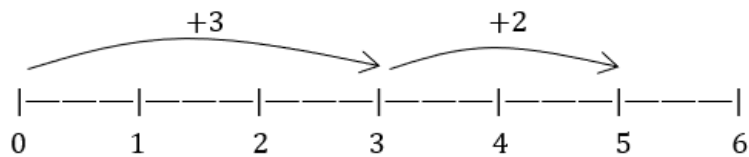
Here are some samples. (Transcriptions are not shown.)

#### Example 9-6

$$(2x + 3)(x - 5) = 2x^2 - 10x + 3x - 15$$

These "pointing" arrows should be drawn as a raised-line tactile graphic.

#### Example 9-7



Methods for depicting arrows above number lines are given in *Guidelines and Standards for Tactile Graphics*.

#### Example 9-8

$$\frac{3}{4} \leftarrow \text{fraction line}$$

The lead line in this diagram could be omitted without a loss of information.



## 9.15 Separation Line

The line printed between the addends and the sum or between the subtrahend and the difference (signifying "equals") is called the "separation line" and is transcribed as a line of dots 25. In braille, the separation line must be made one cell longer at either end than the overall width of the arrangement.

$\cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot$ Separation Line (varying in length)
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## 9.16 Alignment with Addition and Subtraction

In spatial arrangements for addition and subtraction, corresponding digits, commas, and decimal points are vertically aligned – digits under digits, commas under commas, decimal points under decimal points, fractions under fractions, abbreviations under abbreviations, operation signs under operation signs.

*The Examples* Placement of code switch indicators with spatial arrangements is discussed later in this lesson. Assume Nemeth context in these examples. Switch indicators and the blank line before and after each example are not shown.

### Example 9-9

$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$	$\begin{array}{r} \cdot\cdot \\ \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \end{array}$	$\begin{array}{r} 7 \\ -6 \\ \hline 1 \end{array}$	$\begin{array}{r} \cdot\cdot \\ \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \end{array}$
--	--	--	--

*Numeric indicators are not used. Digits are aligned by place value, as printed. The separation line extends one cell to the left and one cell to the right of the overall arrangement.*

### Example 9-10

$\begin{array}{r} 2,017 \\ +1,963 \\ \hline 3,980 \end{array}$	$\begin{array}{r} \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \end{array}$	$\begin{array}{r} 7.5 \\ -6.7 \\ \hline 0.8 \end{array}$	$\begin{array}{r} \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \end{array}$
--	---	--	---

*Commas and decimal points are aligned as printed. The separation line extends one cell to the left and one cell to the right of the overall arrangement.*

- 9.16.1 **Intentional Misalignment.** If items have been intentionally misaligned as an exercise for the student, the misalignment is preserved in the transcription. In [Example 9-11](#) the student has been instructed to rearrange the digits to show proper place value.

*Example 9-11*

$$\begin{array}{r}
 25.92 \\
 10.4 \\
 + 3.796 \\
 \hline
 \end{array}$$

*The decimal points are intentionally misaligned. The braille transcription matches the print arrangement.*

- 9.16.2 **Operation Sign is Absent.** The plus sign is not always present in addition problems. If there is no operation sign, examine the surrounding text to determine that this is indeed an addition problem. Then apply alignment rules for addition.

*Example 9-12*

$$\begin{array}{r}
 .36 \\
 7.02 \\
 3.04 \\
 \hline
 \end{array}$$

## 9.17 Placement of Symbols

- 9.17.1 **Operation Symbols.** The plus or minus symbol indicating addition or subtraction is placed one column of cells to the left of the leftmost numeric symbol in the part of the arrangement which lies above the separation line, regardless of print layout. A space may need to be inserted in order to achieve this alignment.

*Example 9-13*

$  \begin{array}{r}  97 \\  +1 \\  \hline  \end{array}  $	$  \begin{array}{r}  97 \\  +1 \\  \hline  98  \end{array}  $
---	---

*The plus sign is printed within the column reserved for the tens place. In braille, the plus sign is placed one column to the left of the leftmost numeric symbol that appears above the separation line.*

Example 9-14

$$\begin{array}{r}
 877 \\
 -16 \\
 \hline
 861
 \end{array}$$

*The minus sign is printed under the "8" in the minuend. In braille, the minus sign is placed one column to the left of the leftmost numeric symbol that appears above the separation line.*

- a. **Considerations Below the Separation Line.** Placement of the operation symbol is in relation to the numeric characters above the separation line. When an answer is shown below the separation line, a digit may lie in the same column as an operation symbol.

Example 9-15

$$\begin{array}{r}
 65 \\
 +56 \\
 \hline
 121
 \end{array}$$

*Placement of the plus symbol is in relation to the digits above the separation line; the "hundreds" digit in the sum happens to lie in the same column as the plus symbol.*

- 9.17.2 **Monetary Symbols.** Whether shown above or below the separation line, monetary symbols which are printed to the left of values are transcribed at least one column of cells to the left of the leftmost numeric symbol in the part of the arrangement which lies above the separation line. A plus or minus symbol aligned below a monetary symbol is right adjusted in the column. Monetary symbols do not necessarily align.

Example 9-16

$$\begin{array}{r}
 \$ .36 \\
 7.02 \\
 +3.04 \\
 \hline
 \end{array}$$

*The dollar symbol is placed to the left of the leftmost numeric symbol which appears above the separation line. The plus symbol is aligned below the second cell of the dollar symbol.*

Example 9-17

$$\begin{array}{r}
 \$1.45 \\
 - 1.05 \\
 \hline
 \end{array}$$

*The minus sign is printed to the left of the dollar sign. This alignment is not duplicated in braille.*

Example 9-18

$$\begin{array}{r}
 \$ .36 \\
 7.02 \\
 + 3.04 \\
 \hline
 \$10.42
 \end{array}$$

*Each dollar symbol is placed to the left of the leftmost numeric symbol which appears above the separation line. Alignment of the dollar symbols is not a consideration.*

Example 9-19

$$\begin{array}{r}
 \$1.45 \\
 - 1.05 \\
 \hline
 \$ .40
 \end{array}$$

*A space is inserted after the dollar symbol that lies below the separation line in order to align it properly--to the left of the leftmost numeric symbol which appears above the separation line. Alignment of the dollar symbols is coincidental.*

**9.18 Side-by-Side Layout**

When there are two or more spatial problems, it is preferred that they be transcribed side by side across the page. The leftmost symbol, which is often the first symbol of a separation line, is placed in the appropriate cell following the rules for itemized, unitemized, or displayed material, which will be demonstrated in the examples in this section.

In side-by-side layout, care must be taken to leave at least three blank cells between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement. "Any symbol" does not refer to a separation line. In a typical arrangement of similar problems,

this will mean that one blank space will be left between the end of one separation line and the beginning of the next.

Example 9-20

$$\begin{array}{r} 35 \\ -24 \\ \hline \end{array} \quad \begin{array}{r} 975 \\ +25 \\ \hline \end{array} \quad \begin{array}{r} 12,618 \\ -1,414 \\ \hline \end{array} \quad \begin{array}{r} 17.17 \\ +.17 \\ \hline \end{array}$$

*Each separation line extends one cell to the left and one cell to the right of its overall arrangement. One clear column of blank space is between the ends of the separation lines.*

- 9.18.1 **Top Alignment.** The minuend or the first addend in each problem is transcribed on the same line, regardless of the print arrangement. When the separation lines are not on the same braille line, at least one clear column of blank space (one cell's width) must be maintained between the ends of the separation lines. This ensures that at least three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring spatial arrangement (separation lines excluded).

Example 9-21

$$\begin{array}{r} \$3.00 \\ - .99 \\ \hline \end{array} \quad \begin{array}{r} \$7.39 \\ + .25 \\ \hline \end{array} \quad \begin{array}{r} \$.56 \\ .37 \\ \hline \end{array} \quad \begin{array}{r} \$6.26 \\ 5.83 \\ \hline \end{array}$$

*In print, the problems are arranged so that all separation lines are across the same row. In braille, the minuend and addends are top adjusted. One clear column of blank space is between the ends of the separation lines, even those that do not occur on the same braille line.*

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**PRACTICE 9D**

*Instructions:* Placement of code switches and blank lines has not yet been discussed. Please follow these directives in your transcription of this PRACTICE. Transcribe the opening Nemeth Code indicator on a line by itself, in cell 1. The next line should be blank. Begin the problems on the line following the blank line. Use side-by-side layout, with the first cell of the leftmost separation line in cell 1. Leave only one blank line between the two sets of problems. After completion of the last problem, leave one line blank and transcribe a Nemeth Code terminator in cell 1.

900	4391	25.763	718	5,025
<u>+ 100</u>	<u>+ 81</u>	<u>- 4.239</u>	<u>- 437</u>	<u>+ 3,999</u>
1000	4472	21.524	281	9,024

\$ .27	\$3,854	
<u>6.13</u>	<u>602</u>	\$14.99
<u>2.15</u>	<u>+ 918</u>	<u>- 6.37</u>
		\$ 8.62

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### 9.19 Omissions in Work Arranged Spatially for Computation

In a linear problem, the transcriber indicates the omission of an answer by transcribing a general omission symbol. (See Lesson 1.)

$$\gg 123 + 456 = \quad \begin{array}{cccccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array} \quad \begin{array}{cc} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array}$$

On the other hand, when nothing is printed below the separation line in a spatially arranged problem, the transcriber follows print. Do not insert a general omission symbol to represent blank space below a spatially arranged problem.

$$\gg \begin{array}{r} 123 \\ + 456 \\ \hline \end{array} \quad \begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

An omission may be printed any number of ways – as a question mark, an underscore, a box, etc. In braille, only the general omission symbol is used to show an omission within a spatial arrangement. The one-cell symbol allows the transcription to maintain alignment of the place values.

 General Omission Symbol
---

#### Example 9-22

$$\begin{array}{r} 94 \\ + ? \\ \hline 100 \end{array} \quad \begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

*The omission is printed as a questions mark.*

9.19.1 The number of general omission symbols transcribed reflects the number of omission signs used in print.

#### Example 9-23

$$\begin{array}{r} 946 \\ + ?? \\ \hline 1002 \end{array} \quad \begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}$$

*In print, each omission is denoted as a questions mark.*

9.19.2 When only one sign of omission is used in print, only one general omission symbol is transcribed. Follow print for placement of the symbol. When the print sign spans more than one column in the arrangement, align the general omission symbol beneath the rightmost character of the omission.

*Example 9-24*

$$\begin{array}{r} \$7.18 \\ - \quad ? \\ \hline \$5.20 \end{array} \qquad \begin{array}{r} \$7.18 \\ -1.98 \\ \hline \$5.\_ \end{array} \qquad \begin{array}{r} \$\square \\ -1.98 \\ \hline \$5.20 \end{array}$$

*The first omission is printed as a question mark aligned with the decimal point. The second omission is printed as an underscore beneath the decimal digits. (A multipurpose indicator is required between the decimal and the general omission symbol. This will be covered in detail in Lesson 13.) The third omission is printed as a rectangle above the subtrahend.*

## 9.20 Spatially Arranged Polynomials

In polynomials arranged spatially for addition or subtraction, vertical alignment is important.

9.20.1 **Alignment.** Terms, symbols, and indicators are aligned.

*Example 9-25*

$$\begin{array}{r} 5r + 16s + 17t \\ - 4r - 3s \\ \hline r + 13s + 17t \end{array}$$

*Example 9-26*

$$\begin{array}{r} 3x^2 + 4xy + 2 \\ x^2 + 2xy - 15 \\ \hline 4x^2 + 6xy - 13 \end{array}$$

*Level indicators align, as well as coefficients and variables.*



Both polynomials are right justified in print. The student is instructed to align the terms and so the print layout is duplicated in braille. (This example illustrates the proper placement of switch indicators and margins for displayed spatial material. Both topics are discussed later in this lesson.)

## 9.21 Abbreviations

Abbreviations which occur in a spatially arranged addition or subtraction problem are vertically aligned and follow the rules for abbreviations outlined in Lessons 3 and 4.

### Example 9-30

$$\begin{array}{r}
 4 \text{ ft. } 7 \text{ in.} \\
 + 3 \text{ ft. } 5 \text{ in.} \\
 \hline
 7 \text{ ft. } 12 \text{ in.}
 \end{array}$$

## 9.22 Fractions

In a spatial addition or subtraction arrangement containing fractions, the fraction lines and fraction indicators are vertically aligned throughout the problem. Also, each numerator and denominator is in contact with its fraction line, regardless of the number of digits. (Digits in numerators and denominators do not align by place value.) Indicators and numbers must not appear in the same column.

Spaces may need to be inserted to achieve alignment.

### Example 9-31

$$\begin{array}{r}
 1/4 \\
 + 2/4 \\
 \hline
 \end{array}$$

*Fraction indicators are not transcribed when a fraction is printed in this manner. (Section 8.4.2 in Lesson 8)*

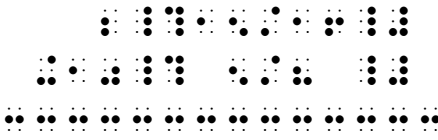
### Example 9-32

$$\begin{array}{r}
 13/16 \\
 + 5/16 \\
 \hline
 \end{array}$$

*The opening and closing fraction indicators are aligned. The fraction lines are aligned. A space is inserted before the 1-digit numerator in order to have it touch the fraction line.*

9.22.1 **Alignment with Mixed Numbers.** The whole number part of a mixed number is vertically aligned by place value.

*Example 9-33*

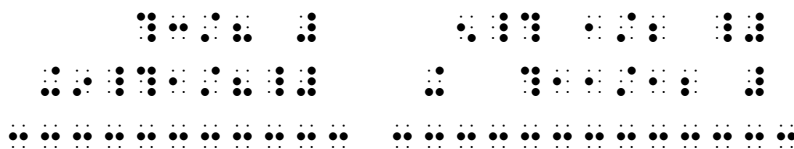
$$\begin{array}{r} 2 \frac{15}{16} \\ + 10 \frac{5}{8} \\ \hline \end{array}$$


*Whole numbers 2 and 10 align by place value.*

9.22.2 **Alignment of Different Fraction Indicators.** When different kinds of fraction indicators occur in the same problem, align the (1456) and (3456) symbols. A space is inserted in order to achieve alignment.

*Example 9-34*

$$\begin{array}{r} \frac{3}{8} \\ + 9 \frac{1}{8} \\ \hline \end{array} \qquad \begin{array}{r} 5 \frac{1}{2} \\ + \frac{11}{12} \\ \hline \end{array}$$



*Notice the blank cell above or below the (456) part of the mixed number indicators to ensure that indicators and numbers do not appear in the same column. Reminder: Each numerator and denominator is in contact with its fraction line.*

9.22.3 **Alignment with the Answer**

The fraction indicators and fraction lines in the answer below the separation line must also align with those above the separation line. Blank cells are inserted in order to achieve alignment.

Example 9-35

$$\begin{array}{r} 12 \\ 16 \\ - 3 \\ \hline 16 \\ 9 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 15 \\ 16 \\ + 5 \\ \hline 8 \\ ? \\ \hline 16 \end{array}$$

$$\begin{array}{r} 9 \frac{2}{4} \\ + 6 \frac{1}{4} \\ \hline 15 \frac{3}{4} \end{array}$$

*Fraction indicators and fraction lines are aligned. Each numerator and denominator is in contact with its fraction line. Take particular notice of the single-digit numerators and denominators as well as the general omission symbol in the second problem. Recall from [Section 9.17.1.a](#) that the plus sign in the third problem can be in the same column as the tens place of the 2-digit whole number in the answer.*

9.22.4 **A Linear Portion of a Spatial Arrangement.** An equals sign printed within a spatial arrangement follows linear spacing rules for signs of comparison.

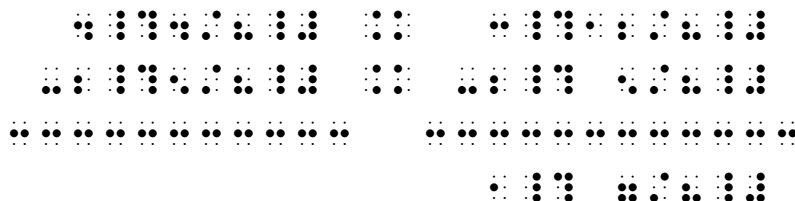
Example 9-36

$$\begin{array}{r} 1 \frac{5}{12} \\ + 6 \frac{5}{12} \\ \hline 7 \frac{10}{12} = 7 \frac{5}{6} \end{array}$$

*A numeric indicator is required for the final answer because it is not part of the vertical computation.*

Example 9-37

$$\begin{array}{r}
 4 \frac{4}{8} = 3 \frac{12}{8} \\
 - 2 \frac{5}{8} = - 2 \frac{5}{8} \\
 \hline
 1 \frac{7}{8}
 \end{array}$$



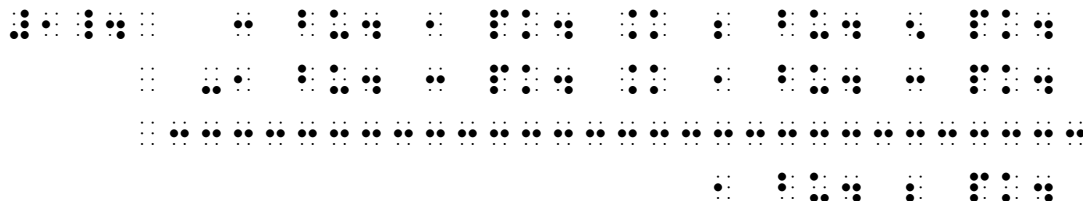
*The equals signs show a relationship between the two side-by-side spatial arrangements.*

**9.23 Placement of Identifiers**

The numeric or alphabetic identifier associated with a spatially arranged addition or subtraction problem is positioned at the top line of the problem—that is, with the first addend or with the subtrahend. One clear column of blank space (one cell’s width) must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines.

Example 9-38

$$\begin{array}{r}
 1. \quad 3 \text{ bu. } 1 \text{ pk.} = 2 \text{ bu. } 5 \text{ pk.} \\
 \quad - 1 \text{ bu. } 3 \text{ pk.} = 1 \text{ bu. } 3 \text{ pk.} \\
 \hline
 \quad \quad \quad 1 \text{ bu. } 2 \text{ pk.}
 \end{array}$$



9.23.1 **Side-by-Side Layout.** Spatially arranged itemized material may be arranged side by side. The identifiers are located on the same braille line across the width of the page, in sequential order. Problems are spaced so that no less than three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement (separation lines excluded) including the neighboring identifier.

Note that this is a special rule pertaining to spatially arranged itemized material. As taught in Lesson 1, each main item in *nonspatial* itemized material must start in cell 1, even if the print copy arranges the items side by side across the page.

Example 9-39

$$\begin{array}{cccccc}
 1) \quad 15 & 2) \quad 15 & 3) \quad 15 & 4) \quad 15 & 5) \quad 150 & 6) \quad 150 \\
 \quad \quad \underline{+3} & \quad \quad \underline{-3} & \quad \quad \underline{+13} & \quad \quad \underline{-13} & \quad \quad \underline{+130} & \quad \quad \underline{-130}
 \end{array}$$

10

11

12

13

14

15

16

17

18

*Lines 11 and 15: Three blank cells come between the rightmost symbol in each problem (the "ones" column) and the numeric indicator of the next identifier.*

*Lines 11-13 and 15-17: One blank cell comes between the right parenthesis of each identifier and the leftmost cell of the separation line on line 3.*

*Line 14: A blank line precedes the second group of spatially arranged problems.*

*Line 15: No attempt is made to align identifiers with those on line 11.*

---

**PRACTICE 9E**

*Instructions:* Transcribe the opening Nemeth Code indicator on a line by itself, in cell 1. The next line should be blank. Begin the problems on the line following the blank line. Use side-by-side layout. After completion of the last problem, leave one line blank and transcribe a Nemeth Code terminator in cell 1.

$$1) \begin{array}{r} 621 \\ + ??? \\ \hline 1096 \end{array}$$

$$2) \begin{array}{r} 17x - 8y - z \\ - 2x + 17y + 6z \\ \hline 15x + 9y + \_ \end{array}$$

$$3) \begin{array}{r} 3x^2 - 5x + 4 \\ - 5x^2 + 12x - 12 \\ \hline -2x^2 + 7x - 8 \end{array}$$

$$4) \begin{array}{r} 1 \frac{2}{3} \text{ yr} \\ + 5 \frac{5}{12} \text{ yr} \\ \hline \end{array}$$

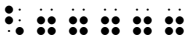
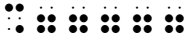
$$6 \frac{13}{12} \text{ yr} = 7 \frac{1}{12} \text{ yr}$$


---

## 9.24 Regrouping Numbers in Addition Problems

Small numbers printed above or below digits in a problem are referred to as "regrouping numbers". (Older texts may call them "carried numbers".) The transcription places the regrouping numbers in the same location as printed, inserting the appropriate regrouping indicator between the regrouping numbers and their associated digits. A regrouping line usually does not appear in print, since the smaller font is enough to distinguish them.

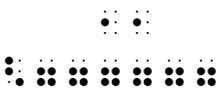
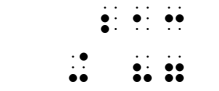
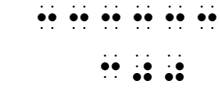
The first cell of the regrouping indicator tells the reader to read the numbers above the line or below the line.

	Regrouping Indicator for Numbers Above the Arrangement (varying in length)
	Regrouping Indicator for Numbers Below the Arrangement (varying in length)

The regrouping indicator will be one cell longer on the left than the separation line. It is important to place the regrouping numbers in the same columnar positions as in print.

In [Example 9-40](#), the regrouping numbers are printed above the first addend. In [Example 9-41](#), the regrouping numbers are printed above the separation line.

### *Example 9-40*

<sup>1</sup> 213	
+ 87	
300	

*In the print copy, the regrouping numbers are in a smaller typeface and a regrouping line is not present.*

Example 9-41

7 9	⠠⠠⠠⠠
2 1 4	⠠⠠⠠⠠⠠
+ 1 8	⠠⠠⠠⠠⠠
<u>3 1 1</u>	⠠⠠⠠⠠⠠⠠⠠
	⠠⠠⠠
	⠠⠠⠠⠠⠠⠠⠠
	⠠⠠⠠

*In the print copy, the regrouping numbers are in a smaller typeface and a regrouping line is not present.*

- 9.24.1 **With Identifiers.** If an identifier is present, it is placed on the first line of the addition problem (the first addend) regardless of the presence of regrouping numbers. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines and regrouping indicators.

Example 9-42

1.	<sup>2</sup> 25.9	⠠⠠⠠⠠⠠
	49.4	⠠⠠⠠⠠⠠
	<u>87.2</u>	⠠⠠⠠⠠⠠
	162.5	⠠⠠⠠⠠⠠⠠⠠
		⠠⠠⠠⠠⠠⠠

## Introduction to Cancellation

### [NC Rule 12]

A spatial arrangement is required when numbers, letters, abbreviations, or words in a mathematical expression are canceled in print by any type of stroke through them, and replacement values are shown. The opening and closing cancellation indicators enclose the material being canceled.

⠠	Opening Cancellation Indicator
⠡	Closing Cancellation Indicator

➤	<del>6</del>	⠠⠠⠡
➤	<del>h</del>	⠠⠠⠡
➤	<del>ft</del>	⠠⠠⠠⠡
➤	<del>mol</del>	⠠⠠⠠⠡

### 9.25 Cancellation in Subtraction Problems

When cancellation is shown in subtraction, a regrouping indicator is not needed. Each regrouping number is aligned with the corresponding canceled digit in the problem.

Digits, decimal points, or other symbols lying above and below the line with cancellation must be carefully aligned for computation. Spaces are inserted where it is necessary to achieve alignment. Numerals or other symbols lying above *and* below the separation line must not appear in the same columns as an indicator.

#### *Example 9-43*

<sup>4</sup> 1	<sup>10</sup> <del>0</del>	⠠	⠠	⠡	⠡
<del>1</del>	<del>0</del>	⠠	⠠	⠡	⠡
-	39	⠠	⠠	⠡	⠡
—		⠠	⠠	⠡	⠡
1	11	⠠	⠠	⠡	⠡

*Place values align throughout the arrangement, including the regrouping numbers. Spaces are inserted to ensure a clear column above and below the cancellation indicators.*

Example 9-44

$$\begin{array}{r}
 \overset{11}{7} \cancel{8} \overset{13}{\cancel{2} \cancel{3}} \\
 - 254 \\
 \hline
 569
 \end{array}$$

*Cancellation indicators align with cancellation indicators. The minus sign on line 4 is placed one column to the left of the opening cancellation indicator on line 3 to ensure a clear column below that indicator.*

Example 9-44 is shown again in order to illustrate placement of decimals and dollar signs in a money problem.

$$\begin{array}{r}
 \overset{11}{7} \cancel{8} \overset{13}{\cancel{2} \cancel{3}} \\
 - 2.54 \\
 \hline
 \$5.69
 \end{array}$$

*The decimal point is not included in the cancellation. Decimal points are aligned throughout the problem. A space is inserted after the dollar sign in the answer in order to keep the column below the cancellation indicator clear. Alignment of the dollar symbols is coincidental.*

**9.25.1 With Identifiers.** If an identifier is present, it is placed on the first line of the subtraction problem (the minuend) regardless of the presence of canceled items. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 9-45

A.

$$\begin{array}{r}
 \overset{7}{\cancel{8}} \overset{12}{\cancel{2}} \overset{4}{\cancel{3}} \overset{16}{\cancel{6}} \\
 - 2347 \\
 \hline
 5909
 \end{array}$$

---

**PRACTICE 9F****Regrouping Numbers and Cancellation**

$$\begin{array}{r} \text{1.} \quad \overset{1}{9} \overset{1}{4} 8 \\ + 75 \\ \hline 1023 \end{array}$$

$$\begin{array}{r} \text{2.} \quad \$ \overset{12}{2} \overset{2}{.} 52 \\ \quad \quad 65.89 \\ \quad \quad 91.99 \\ \quad \quad \underline{32.03} \\ \quad \$192.43 \end{array}$$

$$\begin{array}{r} \text{3.} \quad \overset{4}{\cancel{5}} \overset{16}{\cancel{6}} \overset{1}{\cancel{2}} \overset{13}{\cancel{3}} \\ - 804 \\ \hline 4819 \end{array}$$

## *Arrangement on the Page*

A spatial arrangement should be confined to one braille page. Strategies for larger arrangements are beyond the scope of this course. *The discussion in Sections 9.26 and 9.27 assumes that code switch indicators are not needed, imagining that Nemeth material precedes and follows each example.*

### 9.26 Blank Lines

A blank line is required above and below a spatial arrangement in most situations.

9.26.1 **Blank Lines and the Page Change Indicator.** The presence of a page change indicator does not affect the blank line requirement. There may be times when a blank line is needed both before and after the page change indicator, for example, when spatial arrangements occur both before and after the page change.

*Example 9-46*

15	15
<u>+ 13</u>	<u>- 13</u>

===== new print page 123

150	150
<u>+ 130</u>	<u>- 130</u>

9					
10	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 20px;">⠠⠑⠤</td> <td>⠠⠑⠤</td> </tr> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> </table>	⠠⠑⠤	⠠⠑⠤	⠠⠃⠠⠑	⠠⠃⠠⠑
⠠⠑⠤	⠠⠑⠤				
⠠⠃⠠⠑	⠠⠃⠠⠑				
11	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> </table>	⠠⠃⠠⠑	⠠⠃⠠⠑	⠠⠃⠠⠑	⠠⠃⠠⠑
⠠⠃⠠⠑	⠠⠃⠠⠑				
⠠⠃⠠⠑	⠠⠃⠠⠑				
12	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> </table>	⠠⠃⠠⠑	⠠⠃⠠⠑	⠠⠃⠠⠑	⠠⠃⠠⠑
⠠⠃⠠⠑	⠠⠃⠠⠑				
⠠⠃⠠⠑	⠠⠃⠠⠑				
13					
14	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 20px;">⠠⠑⠤</td> <td>⠠⠑⠤</td> </tr> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> </table>	⠠⠑⠤	⠠⠑⠤	⠠⠃⠠⠑	⠠⠃⠠⠑
⠠⠑⠤	⠠⠑⠤				
⠠⠃⠠⠑	⠠⠃⠠⠑				
15					
16	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 20px;">⠠⠑⠤</td> <td>⠠⠑⠤</td> </tr> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> </table>	⠠⠑⠤	⠠⠑⠤	⠠⠃⠠⠑	⠠⠃⠠⠑
⠠⠑⠤	⠠⠑⠤				
⠠⠃⠠⠑	⠠⠃⠠⠑				
17	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> </table>	⠠⠃⠠⠑	⠠⠃⠠⠑	⠠⠃⠠⠑	⠠⠃⠠⠑
⠠⠃⠠⠑	⠠⠃⠠⠑				
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18	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> <tr> <td style="padding-right: 20px;">⠠⠃⠠⠑</td> <td>⠠⠃⠠⠑</td> </tr> </table>	⠠⠃⠠⠑	⠠⠃⠠⠑	⠠⠃⠠⠑	⠠⠃⠠⠑
⠠⠃⠠⠑	⠠⠃⠠⠑				
⠠⠃⠠⠑	⠠⠃⠠⠑				
19					

*Each set of spatial problems in this example requires a blank line before and after it.*

9.26.2 **Blank Lines and Box Lines.** A blank line is left both above and below a spatial arrangement even if the arrangement directly precedes or follows a box line. (No examples are given.)

9.26.3 **Pagination and Blank Lines.** Transition to a new braille page before beginning or after ending the transcription of a spatial arrangement takes the place of the required blank line.

- a. **Starting a Braille Page with a Spatial Arrangement.** When no running head is in use, a spatial arrangement may begin on line 1. There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the print page number. If a running head is present, line 2 must be blank and the spatial arrangement will begin on line 3.

Example 9-47

1) $\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$	2) $\begin{array}{r} 155 \\ +3 \\ \hline \end{array}$	3) $\begin{array}{r} 155 \\ +133 \\ \hline \end{array}$	
--	---	---	--

1				
2				
3				
4				

*The example above has no running head. The problems begin on line 1.*

Example 9-48

1) $\begin{array}{r} 1 \\ +1 \\ \hline \end{array}$	2) $\begin{array}{r} 2 \\ +2 \\ \hline \end{array}$	3) $\begin{array}{r} 13 \\ +3 \\ \hline \end{array}$	4) $\begin{array}{r} 14 \\ +4 \\ \hline \end{array}$	
---	---	--	--	--

1				
2				
3				
4				
5				
6				

*The example above has a running head. Line 2 is blank.*

- b. **Ending a Braille Page with a Spatial Arrangement.** A spatial arrangement may be placed on the last line of the braille page. There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the braille page number.





## 9.28 Itemized Spatial Problems with Subdivisions

9.28.1 **Without Narrative.** When a set of spatial arrangements contains both main divisions and subdivisions, the following must be observed. If there is no material between the main division identifier and the first subdivision identifier, place the main division at the margin in cell 1, and place the first subdivision on the same braille line after one blank cell. As many subdivisions as possible may be placed across the line. The identifiers are placed as outlined in Sections [9.23](#), [9.24.1](#), and [9.25.1](#), and must be located on the same braille line across the width of the page. If additional subdivisions remain after the width of the page has been met, insert a blank line below the last line of the arrangements. Then begin the next subdivision identifier in cell 3.

*Example 9-52*

$$1. \quad a. \quad \begin{array}{r} 15 \\ +3 \\ \hline \end{array} \quad b. \quad \begin{array}{r} 15 \\ -3 \\ \hline \end{array}$$

*Example 9-53*

$$1. \quad a. \quad \begin{array}{r} 15 \\ +3 \\ \hline \end{array} \quad b. \quad \begin{array}{r} 15 \\ -3 \\ \hline \end{array} \quad c. \quad \begin{array}{r} 15 \\ +13 \\ \hline \end{array} \quad d. \quad \begin{array}{r} 15 \\ -13 \\ \hline \end{array}$$







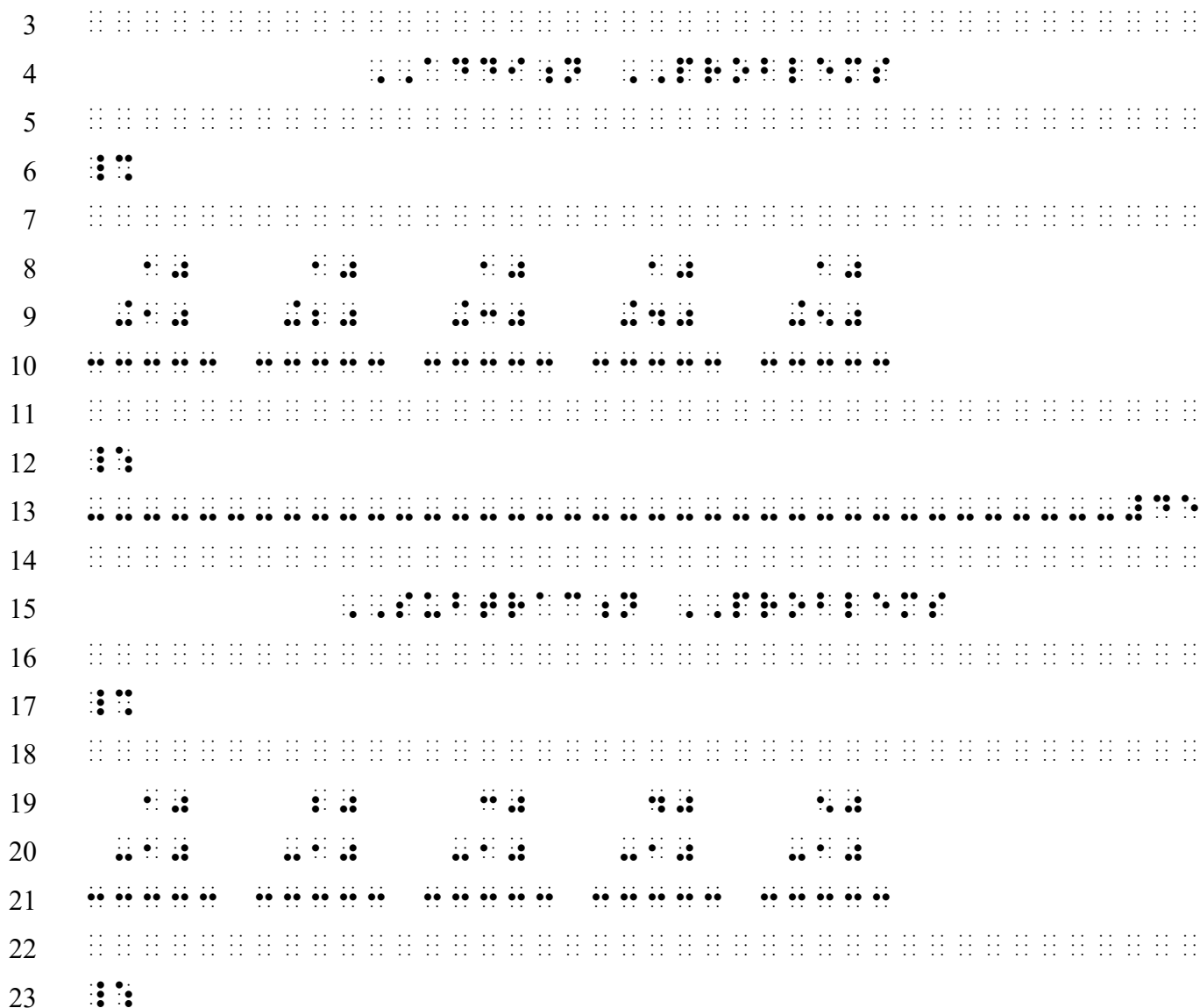












*Line 5: A blank line is required following the centered heading on line 4.*

*Line 6: The opening Nemeth Code indicator is placed in cell 1 on a line by itself.*

*Line 7: A blank line precedes the spatial problem set.*

*Line 11: A blank line follows the spatial problems.*

*Line 12: The Nemeth Code terminator is placed in cell 1 on a line by itself.*

*Line 14: A blank line precedes the centered heading on line 15.*

*Lines 16-23: Same comments as lines 5-12.*

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*For further practice, see Addendum 1—Reading Practice.*

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Submit Exercise 9 to your instructor.









